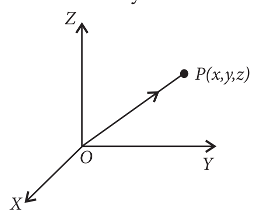
CHAPTER 4

# Vector Algebra

## Recap Notes

### VECTOR

- A physical quantity having magnitude as well as direction is called a vector. A vector is represented by a line segment, denoted as $\overrightarrow{AB}$ or $\vec{a}$. Here, point $A$ is the initial point and $B$ is the terminal point of the vector $\overrightarrow{AB}$.
- **Magnitude :** The distance between the points $A$ and $B$ is called the magnitude of the directed line segment $\overrightarrow{AB}$. It is denoted by $|\overrightarrow{AB}|$.
- **Position Vector :** Let $P$ be any point in space, having coordinates $(x, y, z)$ with respect to some fixed point $O$ (0, 0, 0) as origin, then the vector $\overrightarrow{OP}$ having $O$ as its initial point and $P$ as its terminal point is called the position vector of the point $P$ with respect to $O$. The vector $\overrightarrow{OP}$ is usually denoted by $\vec{r}$.




Magnitude of $\overrightarrow{OP}$ is, $|\overrightarrow{OP}| = \sqrt{x^2 + y^2 + z^2}$

*i.e.*, $|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$.

In general, the position vectors of points $A$, $B$, $C$, etc. with respect to the origin $O$ are denoted by $\vec{a}, \vec{b}, \vec{c}$, etc. respectively.

- **Direction Cosines and Direction Ratios :**
  The angles $\alpha$, $\beta$, $\gamma$ made by the vector $\vec{r}$ with the positive directions of $x$, $y$ and $z$-axes respectively are called its direction angles. The cosine values of these angles, *i.e.*, $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are called direction cosines of the vector $\vec{r}$, and usually denoted by $l$, $m$ and $n$ respectively.
  Direction cosines of $\vec{r}$ are given as

  $$l = \frac{x}{\sqrt{x^2 + y^2 + z^2}}, m = \frac{y}{\sqrt{x^2 + y^2 + z^2}} \text{ and}$$

  $$n = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

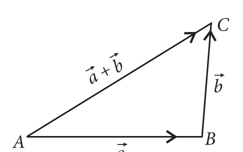
  The numbers $lr$, $mr$ and $nr$, proportional to the direction cosines of vector $\vec{r}$ are called direction ratios of the vector $\vec{r}$ and denoted as $a$, $b$ and $c$ respectively.
  *i.e.*, $a = lr$, $b = mr$ and $c = nr$
  **Note :** $l^2 + m^2 + n^2 = 1$ and $a^2 + b^2 + c^2 \neq 1$, (in general).

### TYPES OF VECTORS

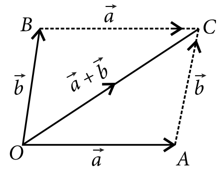
- **Zero vector :** A vector whose initial and terminal points coincide is called a zero (or null) vector. It cannot be assigned a definite direction as it has zero magnitude and it is denoted by the $\vec{0}$.
- **Unit Vector :** A vector whose magnitude is unity *i.e.*, $|\vec{a}| = 1$. It is denoted by $\hat{a}$.
- **Equal Vectors :** Two vectors $\vec{a}$ and $\vec{b}$ are said to be equal, written as $\vec{a} = \vec{b}$, iff they have equal magnitudes and direction regardless of the positions of their initial points.
- **Coinitial Vectors :** Vectors having same initial point are called co-initial vectors.
- **Collinear Vectors :** Two or more vectors are called collinear if they have same or parallel supports, irrespective of their magnitudes and directions.
- **Negative of a Vector :** A vector having the same magnitude as that of a given vector but directed in the opposite sense is called negative of the given vector *i.e.*, $\overrightarrow{BA} = -\overrightarrow{AB}$.

### ADDITION OF VECTORS

- **Triangle law :** Let the vectors be $\vec{a}$ and $\vec{b}$ so positioned such that initial point of one coincides with terminal point of the other. If $\vec{a} = \overrightarrow{AB}$, $\vec{b} = \overrightarrow{BC}$, then the vector $\vec{a} + \vec{b}$ is represented by the third side of $\Delta ABC$ *i.e.*, $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$



- **Parallelogram law :** If the two vectors $\vec{a}$ and $\vec{b}$ are represented by the two adjacent sides $OA$ and $OB$ of a parallelogram $OACB$, then their sum $\vec{a}+\vec{b}$ is represented in magnitude and direction by the diagonal $OC$ of parallelogram $OACB$ through their common point $O$ *i.e.*, $\overrightarrow{OA}+\overrightarrow{OB}=\overrightarrow{OC}$




## Properties of Vector Addition

- Vector addition is commutative *i.e.*, $\vec{a}+\vec{b}=\vec{b}+\vec{a}$.
- Vector addition is associative *i.e.*, $\vec{a}+(\vec{b}+\vec{c})=(\vec{a}+\vec{b})+\vec{c}$.
- Existence of additive identity : The zero vector acts as additive identity *i.e.*, $\vec{a}+\vec{0}=\vec{a}=\vec{0}+\vec{a}$ for any vector $\vec{a}$.
- Existence of additive inverse : The negative of $\vec{a}$ *i.e.*, $-\vec{a}$ acts as additive inverse *i.e.*, $\vec{a}+(-\vec{a})=\vec{0}=(-\vec{a})+\vec{a}$ for any vector $\vec{a}$.

## MULTIPLICATION OF A VECTOR BY A SCALAR

- Let $\vec{a}$ be a given vector and $\lambda$ be a given scalar (a real number), then $\lambda\vec{a}$ is defined as the multiplication of vector $\vec{a}$ by the scalar $\lambda$. Its magnitude is $|\lambda|$ times the modulus of $\vec{a}$ *i.e.*, $|\lambda\vec{a}|=|\lambda|\,|\vec{a}|$.

  Direction of $\lambda\vec{a}$ is same as that of $\vec{a}$ if $\lambda>0$ and opposite to that of $\vec{a}$ if $\lambda<0$.

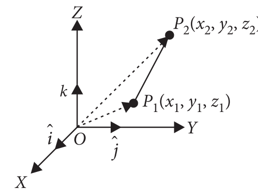
  **Note :** If $\lambda=\dfrac{1}{|\vec{a}|}$, provided that $\vec{a}\neq 0$, then $\lambda\vec{a}$ represents the unit vector in the direction of $\vec{a}$ *i.e.* $\hat{a}=\dfrac{\vec{a}}{|\vec{a}|}$

## COMPONENTS OF A VECTOR

- Let $O$ be the origin and $P(x, y, z)$ be any point in space. Let $\hat{i},\hat{j}$ and $\hat{k}$ be unit vectors along the $X$-axis, $Y$-axis and $Z$-axis respectively. Then $\overrightarrow{OP}=x\hat{i}+y\hat{j}+z\hat{k}$, which is called the component form of $\overrightarrow{OP}$. Here $x$, $y$ and $z$ are scalar components of $\overrightarrow{OP}$ and $x\hat{i}$, $y\hat{j}$ and $z\hat{k}$ are vector components of $\overrightarrow{OP}$.
- If $\vec{a}$ and $\vec{b}$ are two given vectors as $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ and $\lambda$ be any scalar, then
  - (i) $\vec{a}+\vec{b}=(a_1+b_1)\hat{i}+(a_2+b_2)\hat{j}+(a_3+b_3)\hat{k}$
  - (ii) $\vec{a}-\vec{b}=(a_1-b_1)\hat{i}+(a_2-b_2)\hat{j}+(a_3-b_3)\hat{k}$
  - (iii) $\lambda\vec{a}=(\lambda a_1)\hat{i}+(\lambda a_2)\hat{j}+(\lambda a_3)\hat{k}$
  - (iv) $\vec{a}=\vec{b}\Leftrightarrow a_1=b_1, a_2=b_2$ and $a_3=b_3$
  - (v) $\vec{a}$ and $\vec{b}$ are collinear iff $\dfrac{b_1}{a_1}=\dfrac{b_2}{a_2}=\dfrac{b_3}{a_3}=\lambda$.

## VECTOR JOINING TWO POINTS

- If $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ are any two points in the space, then the vector joining $P_1$ and $P_2$ is the vector $\overrightarrow{P_1P_2}$.




  Applying triangle law in $\Delta OP_1P_2$, we get

  $\overrightarrow{OP_1}+\overrightarrow{P_1P_2}=\overrightarrow{OP_2}$

  $\Rightarrow\ \overrightarrow{P_1P_2}=\overrightarrow{OP_2}-\overrightarrow{OP_1}$

  $=(x_2\hat{i}+y_2\hat{j}+z_2\hat{k})-(x_1\hat{i}+y_1\hat{j}+z_1\hat{k})$

  $=(x_2-x_1)\hat{i}+(y_2-y_1)\hat{j}+(z_2-z_1)\hat{k}$

  $\therefore\ |\overrightarrow{P_1P_2}|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

## SECTION FORMULA

- Let $A$, $B$ be two points such that $\overrightarrow{OA}=\vec{a}$ and $\overrightarrow{OB}=\vec{b}$.
- The position vector $\vec{r}$ of the point $P$ which divides the line segment $AB$ internally in the ratio $m : n$ is given by $\vec{r}=\dfrac{m\vec{b}+n\vec{a}}{m+n}$.
- The position vector $\vec{r}$ of the point $P$ which divides the line segment $AB$ externally in the ratio $m : n$ is given by $\vec{r}=\dfrac{m\vec{b}-n\vec{a}}{m-n}$.
- The position vector $\vec{r}$ of the mid-point of the line segment $AB$ is given by $\vec{r}=\dfrac{\vec{a}+\vec{b}}{2}$.

## PRODUCT OF TWO VECTORS

- **Scalar (or dot) product :** The scalar (or dot) product of two (non-zero) vectors $\vec{a}$ and $\vec{b}$, denoted by $\vec{a}\cdot\vec{b}$ (read as $\vec{a}$ dot $\vec{b}$), is defined as $\vec{a}\cdot\vec{b}=|\vec{a}|\,|\vec{b}|\cos\theta=ab\cos\theta$, where, $a=|\vec{a}|, b=|\vec{b}|$ and $\theta(0\le\theta\le\pi)$ is the angle between $\vec{a}$ and $\vec{b}$.
- **Properties of Scalar Product :**
  - (i) Scalar product is commutative : $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$
  - (ii) $\vec{a}\cdot\vec{0}=0$
  - (iii) Scalar product is distributive over addition :

    $\vec{a}\cdot(\vec{b}+\vec{c})=\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}$

    $(\vec{a}+\vec{b})\cdot\vec{c}=\vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{c}$
  - (iv) $\lambda(\vec{a}\cdot\vec{b})=(\lambda\vec{a})\cdot\vec{b}=\vec{a}\cdot(\lambda\vec{b})$, $\lambda$ be any scalar.

(v) If $\hat{i}$, $\hat{j}$ and $\hat{k}$ are three unit vectors along three mutually perpendicular lines, then

$\hat{i}\cdot\hat{i}=\hat{j}\cdot\hat{j}=\hat{k}\cdot\hat{k}=1$ and $\hat{i}\cdot\hat{j}=\hat{j}\cdot\hat{k}=\hat{k}\cdot\hat{i}=0$

(vi) Angle between two non-zero vectors $\vec{a}$ and $\vec{b}$ is given by $\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$.

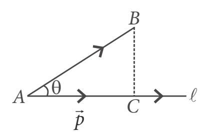
*i.e.*, $\theta = \cos^{-1}\left(\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|}\right)$

(vii) Two non-zero vectors $\vec{a}$ and $\vec{b}$ are mutually perpendicular if and only if $\vec{a}\cdot\vec{b}=0$

(viii) If $\theta = 0$, then $\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|$

If $\theta = \pi$, then $\vec{a}\cdot\vec{b}=-|\vec{a}||\vec{b}|$

➤ **Projection of a vector on a line :**




Let the vector $\overrightarrow{AB}$ makes an angle $\theta$ with directed line $\ell$.

Projection of $\overrightarrow{AB}$ on $\ell$

$=|\overrightarrow{AB}|\cos\theta=\overrightarrow{AC}=\vec{p}.$

The vector $\vec{p}$ is called the projection vector. Its magnitude is $|\vec{p}|$, which is known as projection of vector $\overrightarrow{AB}$.

Projection of a vector $\vec{a}$ on $\vec{b}$, is given as $\vec{a}\cdot\hat{b}$ *i.e.,* $\dfrac{1}{|\vec{b}|}(\vec{a}\cdot\vec{b})$.

➤ **Vector (or Cross) Product :** The vector (or cross) product of two (non-zero) vectors $\vec{a}$ and $\vec{b}$ (in an assigned order), denoted by $\vec{a}\times\vec{b}$ (read as $\vec{a}$ cross $\vec{b}$), is defined as $\vec{a}\times\vec{b}=|\vec{a}|\,|\vec{b}|\sin\theta\,\hat{n}$ where $\theta(0\le\theta\le\pi)$ is the angle between $\vec{a}$ and $\vec{b}$ and $\hat{n}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$.

➤ **Properties of Vector Product :**

(i) Non-commutative : $\vec{a}\times\vec{b}=-\vec{b}\times\vec{a}$

(ii) Vector product is distributive over addition :

$\vec{a}\times(\vec{b}+\vec{c})=\vec{a}\times\vec{b}+\vec{a}\times\vec{c}$

(iii) $\lambda(\vec{a}\times\vec{b})=(\lambda\vec{a})\times\vec{b}=\vec{a}\times(\lambda\vec{b})$, $\lambda$ be any scalar.

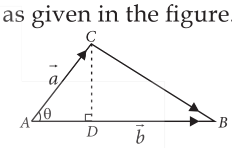
(iv) $(\lambda_1\vec{a})\times(\lambda_2\vec{b})=\lambda_1\lambda_2(\vec{a}\times\vec{b})$

(v) $\hat{i}\times\hat{j}=\hat{k},\ \hat{j}\times\hat{k}=\hat{i},\ \hat{k}\times\hat{i}=\hat{j}$

(vi) Two non-zero vectors $\vec{a}$, $\vec{b}$ are collinear if and only if $\vec{a}\times\vec{b}=\vec{0}$

Similarly, $\vec{a}\times\vec{a}=\vec{0}$ and $\vec{a}\times(-\vec{a})=\vec{0}$, since in the first situation $\theta = 0$ and in the second one, $\theta = \pi$, making the value of $\sin\theta$ to be 0.

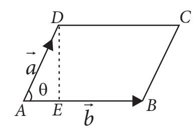
(vii) If $\vec{a}$ and $\vec{b}$ represent the adjacent sides of a triangle as given in the figure. Then,




Area of triangle $ABC = \dfrac{1}{2}AB\cdot CD$

$=\dfrac{1}{2}|\vec{b}|\,|\vec{a}|\sin\theta\ =\dfrac{1}{2}|\vec{a}\times\vec{b}|$

(viii) If $\vec{a}$ and $\vec{b}$ represent the adjacent sides of a parallelogram as given in the figure.




Then, area of parallelogram $ABCD = AB\cdot DE$

$=|\vec{b}||\vec{a}|\sin\theta=|\vec{a}\times\vec{b}|$

(ix) If $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k},\ \vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$,

Then, $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$

$=(a_2b_3-a_3b_2)\hat{i}+(a_3b_1-a_1b_3)\hat{j}+(a_1b_2-a_2b_1)\hat{k}$

(x) Angle between two vectors $\vec{a}$ and $\vec{b}$ is given by

$\sin\theta = \dfrac{|\vec{a}\times\vec{b}|}{|\vec{a}|\,|\vec{b}|}$

*i.e.,* $\theta = \sin^{-1}\left(\dfrac{|\vec{a}\times\vec{b}|}{|\vec{a}|\,|\vec{b}|}\right)$

# Practice Time

## OBJECTIVE TYPE QUESTIONS

### Multiple Choice Questions (MCQs)

**1.** Find the sum of the vectors $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$.

(a) $-4\hat{j} - \hat{k}$ (b) $-\hat{i} - 4\hat{j} - \hat{k}$
(c) $4\hat{j} + \hat{k}$ (d) $\hat{i} - 4\hat{j}$

**2.** The magnitude of the vector $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$ is equal to

(a) 6 (b) 7 (c) 7.5 (d) 8.5

**3.** $(\vec{a} \cdot \hat{i})^2 + (\vec{a} \cdot \hat{j})^2 + (\vec{a} \cdot \hat{k})^2$ is equal to

(a) 1 (b) $|\vec{a}|$ (c) $-\vec{a}$ (d) $|\vec{a}|^2$

**4.** If $ABCD$ is a rhombus, whose diagonals intersect at $E$, then $\overrightarrow{EA} + \overrightarrow{EB} + \overrightarrow{EC} + \overrightarrow{ED}$ equals

(a) $\vec{0}$ (b) $\overrightarrow{AD}$ (c) $2\overrightarrow{BC}$ (d) $2\overrightarrow{AD}$

**5.** If $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 4\hat{i} + 4\hat{j} - 2\hat{k}$ then find the angle between the vectors $\vec{a}$ and $\vec{b}$.

(a) 0 (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

**6.** The projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ is

(a) $\frac{10}{\sqrt{6}}$ (b) $\frac{10}{\sqrt{3}}$ (c) $\frac{5}{\sqrt{6}}$ (d) $\frac{5}{\sqrt{3}}$

**7.** If $A$ and $B$ are the points (– 3, 4, – 8) and (5, – 6, 4) respectively, then find the ratio in which $yz$-plane divides $\overrightarrow{AB}$.

(a) 5 : 2 (b) 7 : 5 (c) 3 : 5 (d) 5 : 3

**8.** The vector in the direction of the vector $\hat{i} - 2\hat{j} + 2\hat{k}$ that has magnitude 9 is

(a) $\hat{i} - 2\hat{j} + 2\hat{k}$ (b) $\frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$
(c) $3(\hat{i} - 2\hat{j} + 2\hat{k})$ (d) $9(\hat{i} - 2\hat{j} + 2\hat{k})$

**9.** If the angle between $\hat{i} + \hat{k}$ and $\hat{i} + \hat{j} + a\hat{k}$ is $\frac{\pi}{3}$ then the value of $a$ is

(a) 0 or 2 (b) –4 or 0
(c) 0 or –2 (d) 2 or –2

**10.** If $|\vec{a} - \vec{b}| = |\vec{a}| = |\vec{b}| = 1$, then the angle between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{\pi}{3}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{6}$

**11.** If $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}, \vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$, then the value of $\vec{a} \cdot (\vec{b} \times \vec{c})$ is

(a) – 20 (b) – 10 (c) 10 (d) 20

**12.** The magnitude of each of the two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude such that the angle between them is 60° and their scalar product is $\frac{9}{2}$, is

(a) 2 (b) 3 (c) 4 (d) 5

**13.** If $\vec{u} = \hat{i} + 2\hat{j}$, $\vec{v} = -2\hat{i} + \hat{j}$ and $\vec{w} = 4\hat{i} + 3\hat{j}$, then find scalars $x$ and such that $\vec{w} = x\vec{u} + y\vec{v}$.

(a) $x = 4, y = -2$ (b) $x = 2, y = -1$
(c) $x = 3, y = 5$ (d) $x = -5, y = 2$

**14.** Write the direction cosines of the vector $-2\hat{i} + \hat{j} - 5\hat{k}$.

(a) $\left(\frac{2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{5}{\sqrt{30}}\right)$

(b) $\left(\frac{-2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{-5}{\sqrt{30}}\right)$

(c) $\left(-\frac{2}{\sqrt{30}}, -\frac{1}{\sqrt{30}}, \frac{-5}{\sqrt{30}}\right)$

(d) None of these

**15.** Let $\vec{a}$ and $\vec{b}$ are non-collinear. If $\vec{c} = (x-2)\vec{a} + \vec{b}$ and $\vec{d} = (2x+1)\vec{a} - \vec{b}$ are collinear, then find the value of $x$.

(a) $\frac{2}{3}$ (b) $\frac{-1}{3}$ (c) $\frac{-2}{3}$ (d) $\frac{1}{3}$

**16.** If $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$, then $|\vec{a} \times \vec{b}|$ is equal to

(a) $\sqrt{507}$ (b) $\sqrt{506}$ (c) $\sqrt{508}$ (d) $\sqrt{509}$

**17.** $(\hat{i}+\hat{j})\times(\hat{j}+\hat{k})\cdot(\hat{k}+\hat{i})$ is equal to

(a) 0 (b) 1 (c) 2 (d) –1

**18.** If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined to $x$-axis at angles 30° and 120° respectively, then $|\vec{a}+\vec{b}|$ equals

(a) $\sqrt{\frac{2}{3}}$ (b) $\sqrt{2}$ (c) $\sqrt{3}$ (d) 2

**19.** If $\vec{b}$ and $\vec{c}$ are any two non-collinear unit vectors and $\vec{a}$ is any vector, then find the value of $(\vec{a}\cdot\vec{b})\vec{b}+(\vec{a}\cdot\vec{c})\vec{c}+\frac{\vec{a}\cdot(\vec{b}\times\vec{c})}{|\vec{b}\times\vec{c}|}\cdot(\vec{b}\times\vec{c})$.

(a) $\vec{a}+\vec{b}+\vec{c}$ (b) $\vec{c}$

(c) $\vec{a}$ (d) $\vec{b}$

**20.** If $\vec{a}$ and $\vec{b}$ are unit vectors enclosing an angle θ and $|\vec{a}+\vec{b}|<1$, then

(a) $\theta=\frac{\pi}{2}$ (b) $\theta<\frac{\pi}{3}$

(c) $\pi\geq\theta>\frac{2\pi}{3}$ (d) $\frac{\pi}{3}<\theta<\frac{2\pi}{3}$

**21.** If $|\vec{a}|=10$, $|\vec{b}|=2$ and $\vec{a}\cdot\vec{b}=12$, then the value of $|\vec{a}\times\vec{b}|$ is

(a) 5 (b) 10 (c) 14 (d) 16

**22.** If $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=3\hat{i}-\hat{j}+2\hat{k}$, then $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are

(a) parallel (b) perpendicular

(c) skew (d) None of these

**23.** Area of a parallelogram whose adjacent sides are represented by the vectors $2\hat{i}-3\hat{k}$ and $4\hat{j}+2\hat{k}$ is

(a) $4\sqrt{14}$ sq. units (b) $2\sqrt{7}$ sq. units

(c) $4\sqrt{7}$ sq. units (d) $4\sqrt{19}$ sq. units

**24.** The direction ratios of the vector $3\vec{a}+2\vec{b}$, where $\vec{a}=\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b}=2\hat{i}-4\hat{j}+5\hat{k}$ are

(a) 7, 5, 4 (b) 7, –5, 4

(c) – 7, 5, 4 (d) 7, 5, – 4

**25.** The angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 4, respectively and $\vec{a}\cdot\vec{b}=2\sqrt{3}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{5\pi}{2}$

**26.** The position vector of the point which divides the joining of points $2\vec{a}-3\vec{b}$ and $\vec{a}+\vec{b}$ in the ratio 3 : 1 is

(a) $\frac{3\vec{a}-2\vec{b}}{2}$ (b) $\frac{7\vec{a}-8\vec{b}}{4}$

(c) $\frac{3\vec{a}}{4}$ (d) $\frac{5\vec{a}}{4}$

**27.** If $|\vec{a}|=4$ and $-3\leq\lambda\leq3$, then the range of $|\lambda\vec{a}|$ is

(a) [0, 8] (b) [–12, 8]

(c) [0, 12] (d) [8, 12]

**28.** If $(2\hat{i}+6\hat{j}+27\hat{k})\times(\hat{i}+p\hat{j}+q\hat{k})=\vec{0}$, then the values of $p$ and $q$ are

(a) $p=6,\ q=27$ (b) $p=3,\ q=\frac{27}{2}$

(c) $p=6,\ q=\frac{27}{2}$ (d) $p=3,\ q=27$

**29.** If $\vec{a},\vec{b},\vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then write the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$.

(a) $\frac{3}{2}$ (b) 3 (c) $\frac{-3}{2}$ (d) – 3

**30.** Find the value of λ for which the vectors $3\hat{i}-6\hat{j}+\hat{k}$ and $2\hat{i}-4\hat{j}+\lambda\hat{k}$ are parallel.

(a) $\frac{2}{3}$ (b) $\frac{-3}{2}$ (c) $\frac{-2}{3}$ (d) $\frac{3}{2}$

**31.** Find the value of λ so that the vectors $2\hat{i}-4\hat{j}+\hat{k}$ and $4\hat{i}-8\hat{j}+\lambda\hat{k}$ are perpendicular.

(a) 20 (b) – 40 (c) 40 (d) – 20

**32.** Find the projection of the vector $\hat{i}+3\hat{j}+7\hat{k}$ on the vector $2\hat{i}-3\hat{j}+6\hat{k}$.

(a) 5 (b) 6 (c) 7 (d) 8

**33.** The vector having initial and terminal points as (2, 5, 0) and (–3, 7, 4) respectively is

(a) $-\hat{i}+12\hat{j}+4\hat{k}$ (b) $5\hat{i}+2\hat{j}-4\hat{k}$

(c) $-5\hat{i}+2\hat{j}+4\hat{k}$ (d) $\hat{i}+\hat{j}+\hat{k}$

**34.** The vectors from origin to the points $A$ and $B$ are $\vec{a}=2\hat{i}-3\hat{j}+2\hat{k}$ and $\vec{b}=2\hat{i}+3\hat{j}+\hat{k}$, respectively, then the area of triangle $OAB$ (in sq. units) is

(a) $\sqrt{340}$ (b) $\sqrt{325}$

(c) $\sqrt{229}$ (d) $\frac{1}{2}\sqrt{229}$

**35.** If $|\vec{a}|=3$, $|\vec{b}|=4$, then the value of λ for which $\vec{a}+\lambda\vec{b}$ is perpendicular to $\vec{a}-\lambda\vec{b}$, is

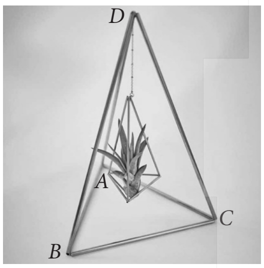
(a) $\frac{9}{16}$ (b) $\frac{3}{4}$ (c) $\frac{3}{2}$ (d) $\frac{4}{3}$

# Case Based MCQs

**Case I :** Read the following passage and answer the questions from 36 to 40.

Ginni purchased an air plant holder which is in the shape of a tetrahedron.

Let $A, B, C$ and $D$ are the coordinates of the air plant holder where $A \equiv (1, 1, 1)$, $B \equiv (2, 1, 3)$, $C \equiv (3, 2, 2)$ and $D \equiv (3, 3, 4)$.




**36.** Find the position vector of $\overrightarrow{AB}$.

(a) $-\hat{i} - 2\hat{k}$ (b) $2\hat{i} + \hat{k}$

(c) $\hat{i} + 2\hat{k}$ (d) $-2\hat{i} - \hat{k}$

**37.** Find the position vector of $\overrightarrow{AC}$.

(a) $2\hat{i} - \hat{j} - \hat{k}$ (b) $2\hat{i} + \hat{j} + \hat{k}$

(c) $-2\hat{i} - \hat{j} + \hat{k}$ (d) $\hat{i} + 2\hat{j} + \hat{k}$

**38.** Find the position vector of $\overrightarrow{AD}$.

(a) $2\hat{i} - 2\hat{j} - 3\hat{k}$ (b) $\hat{i} + \hat{j} - 3\hat{k}$

(c) $3\hat{i} + 2\hat{j} + 2\hat{k}$ (d) $2\hat{i} + 2\hat{j} + 3\hat{k}$

**39.** Area of $\Delta ABC$ =

(a) $\frac{\sqrt{11}}{2}$ sq. units (b) $\frac{\sqrt{14}}{2}$ sq. units

(c) $\frac{\sqrt{13}}{2}$ (d) $\frac{\sqrt{17}}{2}$ sq. units

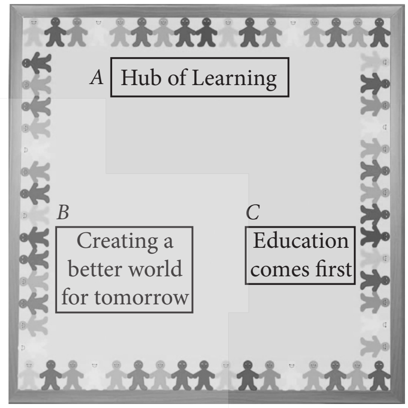
**40.** Find the unit vector along $\overrightarrow{AD}$.

(a) $\frac{1}{\sqrt{17}}(2\hat{i} + 2\hat{j} + 3\hat{k})$ (b) $\frac{1}{\sqrt{17}}(3\hat{i} + 3\hat{j} + 2\hat{k})$

(c) $\frac{1}{\sqrt{11}}(2\hat{i} + 2\hat{j} + 3\hat{k})$ (d) $(2\hat{i} + 2\hat{j} + 3\hat{k})$

**Case II :** Read the following passage and answer the questions from 41 to 45.

Three slogans on chart papers are to be placed on a school bulletin board at the points $A$, $B$ and $C$ displaying $A$ (Hub of Learning), $B$ (Creating a better world for tomorrow) and $C$ (Education comes first). The coordinates of points $A$, $B$ and $C$ are (1, 4, 2), (3, –3, –2) and (–2, 2, 6) respectively.




**41.** Let $\vec{a}, \vec{b}$ and $\vec{c}$ be the position vectors of points $A$, $B$ and $C$ respectively, then $\vec{a} + \vec{b} + \vec{c}$ is equal to

(a) $2\hat{i} + 3\hat{j} + 6\hat{k}$ (b) $2\hat{i} - 3\hat{j} - 6\hat{k}$

(c) $2\hat{i} + 8\hat{j} + 3\hat{k}$ (d) $2(7\hat{i} + 8\hat{j} + 3\hat{k})$

**42** Which of the following is not true?

(a) $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$ (b) $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0}$

(c) $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{CA} = \vec{0}$ (d) $\overrightarrow{AB} - \overrightarrow{CB} + \overrightarrow{CA} = \vec{0}$

**43** Area of $\Delta ABC$ is

(a) 19 sq. units (b) $\sqrt{1937}$ sq. units

(c) $\frac{1}{2}\sqrt{1937}$ sq. units (d) $\sqrt{1837}$ sq. units

**44.** Suppose, if the given slogans are to be placed on a straight line, then the value of $|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$ will be equal to

(a) –1 (b) –2

(c) 2 (d) 0

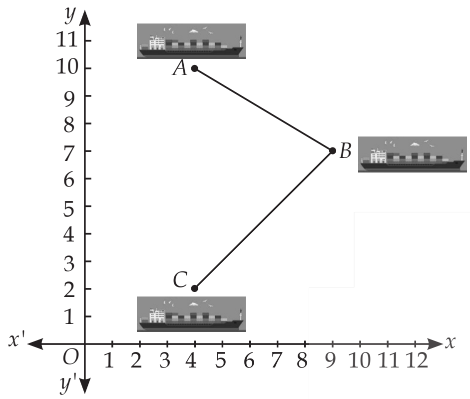
**45.** If $\vec{a} = 2\hat{i} + 3\hat{j} + 6\hat{k}$, then unit vector in the direction of vector $\vec{a}$ is

(a) $\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}$ (b) $\frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$

(c) $\frac{3}{7}\hat{i} + \frac{2}{7}\hat{j} + \frac{6}{7}\hat{k}$ (d) None of these

**Case III :** Read the following passage and answer the questions from 46 to 50.

A barge is pulled into harbour by two tug boats as shown in the figure.




**46.** Position vector of $A$ is

(a) $4\hat{i}+2\hat{j}$ (b) $4\hat{i}+10\hat{j}$

(c) $4\hat{i}-10\hat{j}$ (d) $4\hat{i}-2\hat{j}$

**47.** Position vector of $B$ is

(a) $4\hat{i}+4\hat{j}$ (b) $6\hat{i}+6\hat{j}$

(c) $9\hat{i}+7\hat{j}$ (d) $3\hat{i}+3\hat{j}$

**48.** Find the vector $\overline{AC}$.

(a) $8\hat{j}$ (b) $-8\hat{j}$

(c) $8\hat{i}$ (d) None of these

**49.** If $\vec{A}=\hat{i}+2\hat{j}+3\hat{k}$, then its unit vector is

(a) $\frac{\hat{i}}{\sqrt{14}}+\frac{2\hat{j}}{\sqrt{14}}+\frac{3\hat{k}}{\sqrt{14}}$ (b) $\frac{3\hat{i}}{\sqrt{14}}+\frac{2\hat{j}}{\sqrt{14}}+\frac{\hat{k}}{\sqrt{14}}$

(c) $\frac{2\hat{i}}{\sqrt{14}}+\frac{3\hat{j}}{\sqrt{14}}+\frac{\hat{k}}{\sqrt{14}}$ (d) None of these

**50.** If $\vec{A}=4\hat{i}+3\hat{j}$ and $\vec{B}=3\hat{i}+4\hat{j}$, then $|\vec{A}|+|\vec{B}|=$

(a) 12 (b) 13 (c) 14 (d) 10

## Assertion & Reasoning Based MCQs

**Directions (Q.-51 to 60) :** In these questions, a statement of Assertion is followed by a statement of Reason is given. Choose the correct answer out of the following choices :

*(a) Assertion and Reason both are correct statements and Reason is the correct explanation of Assertion.*
*(b) Assertion and Reason both are correct statements but Reason is not the correct explanation of Assertion.*
*(c) Assertion is correct statement but Reason is wrong statement.*
*(d) Assertion is wrong statement but Reason is correct statement.*

**51. Assertion :** The magnitude of the resultant of vectors $\vec{a}=2\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$ is $\sqrt{34}$.

**Reason :** The magnitude of a vector can never be negative.

**52. Assertion :** The unit vector in the direction of sum of the vectors $\hat{i}+\hat{j}+\hat{k}$, $2\hat{i}-\hat{j}-\hat{k}$ and $2\hat{j}+6\hat{k}$ is $-\frac{1}{7}(3\hat{i}+2\hat{j}+6\hat{k})$.

**Reason :** Let $\vec{a}$ be a non-zero vector, then $\frac{\vec{a}}{|\vec{a}|}$ is a unit vector parallel to $\vec{a}$.

**53.** Let $\vec{a}=\hat{i}+\hat{j}-3\hat{k}$ and $\vec{b}=2\hat{i}+\hat{j}+\hat{k}$.

**Assertion :** Vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other.

**Reason :** $\vec{a}\cdot\vec{b}=0$

**54. Assertion :** The adjacent sides of a parallelogram are along $\vec{a}=\hat{i}+2\hat{j}$ and $\vec{b}=2\hat{i}+\hat{j}$. The angle between the diagonals is 150°.

**Reason :** Two vectors are perpendicular to each other if their dot product is zero.

**55. Assertion :** If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, $|\vec{a}|=3$, $|\vec{b}|=4$, $|\vec{c}|=5$, then $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ is equal to $-25$.

**Reason :** If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then the angle $\theta$ between $\vec{b}$ and $\vec{c}$ is given by $\cos\theta=\frac{\vec{a}^2-\vec{b}^2-\vec{c}^2}{2\vec{b}\,\vec{c}}$.

**56. Assertion :** The length of projection of the vector $3\hat{i}-\hat{j}-2\hat{k}$ on the vector $\hat{i}+2\hat{j}-3\hat{k}$ is $\frac{7}{\sqrt{14}}$.

**Reason :** The projection of a vector $\vec{a}$ on another vector $\vec{b}$ is $\frac{(\vec{a}\cdot\vec{b})}{|\vec{b}|}$.

**57.** Let $\vec{a}$ and $\vec{b}$ be proper vectors and θ be the angle between them.

**Assertion :** $(\vec{a}\times\vec{b})^2 + (\vec{a}\cdot\vec{b})^2 \neq (\vec{a})^2(\vec{b})^2$

**Reason :** $\sin^2\theta + \cos^2\theta = 1$

**58. Assertion :** If $(\vec{a}\times\vec{b})^2 + (\vec{a}\cdot\vec{b})^2 = 400$ and $|\vec{a}| = 4$, then $|\vec{b}| = 9$.

**Reason :** If $\vec{a}$ and $\vec{b}$ are any two vectors, then $(\vec{a}\times\vec{b})^2$ is equal to $(\vec{a})^2(\vec{b})^2 - (\vec{a}\cdot\vec{b})^2$.

**59. Assertion :** If $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}, \vec{b} = -\hat{i} + 3\hat{j} + 4\hat{k}$ then projection of on .

**Reason :** Projection of $\vec{a}$ on $\vec{b} = \dfrac{3}{\sqrt{26}}$.

**60. Assertion:** Three points with position vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are collinear if $\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a} = \vec{0}$

**Reason :** If $\overrightarrow{AB}\cdot\overrightarrow{AC} = 0$, then $\overrightarrow{AB} \perp \overrightarrow{AC}$.

## SUBJECTIVE TYPE QUESTIONS

### Very Short Answer Type Questions (VSA)

**1.** If a unit vector $\vec{a}$ makes angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and an acute angle θ with $\hat{k}$, then find the value of θ.

**2.** Find the sum of the following vectors. $\vec{a} = \hat{i} - 3\hat{k}, \vec{b} = 2\hat{j} - \hat{k}, \vec{c} = 2\hat{i} - 3\hat{j} + 2\hat{k}$

**3.** Find a unit vector in the direction of the sum of the vectors $\vec{a} = 2\hat{i} + 2\hat{j} - 5\hat{k}$ and $\vec{b} = 4\hat{i} - 3\hat{j} + 2\hat{k}$.

**4.** $L$ and $M$ are two points with position vectors $2\vec{a} - \vec{b}$ and $\vec{a} + 2\vec{b}$ respectively. What is the position vector of a point $N$ which divides the line segment $LM$ in the ratio 2 : 1 externally?

**5.** Find the value of $|\vec{a}\times\vec{b}|^2 + |\vec{a}\cdot\vec{b}|^2$ if $|\vec{a}| = 5$ and $|\vec{b}| = 4$.

**6.** Find the area of a parallelogram whose adjacent sides are represented by the vectors $\hat{i} - 3\hat{k}$ and $2\hat{j} + \hat{k}$.

**7.** Find the projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = 2\hat{i} + 2\hat{j} + \hat{k}$.

**8.** If $\vec{a}$ and $\vec{b}$ are unit vectors, then find the angle between $\vec{a}$ and $\vec{b}$, given that $\left(\sqrt{2}\,\vec{a} - \vec{b}\right)$ is a unit vector.

**9.** Find the angle between $x$-axis and the vector $\hat{i} + \hat{j} + \hat{k}$.

**10.** If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a} + \vec{b}| = |\vec{a}|$, then prove that vector $2\vec{a} + \vec{b}$ is perpendicular to vector $\vec{b}$.

### Short Answer Type Questions (SA-I)

**11.** $X$ and $Y$ are two points with position vectors $3\vec{a} + \vec{b}$ and $\vec{a} - 3\vec{b}$ respectively. Write the position vector of a point $Z$ which divides the line segment $XY$ in the ratio 2 : 1 externally.

**12.** Find a unit vector perpendicular to each of the vectors $\vec{a}$ and $\vec{b}$ and where $\vec{a} = 5\hat{i} + 6\hat{j} - 2\hat{k}$ and $\vec{b} = 7\hat{i} + 6\hat{j} + 2\hat{k}$.

**13.** Show that for any two non-zero vectors $\vec{a}$ and $\vec{b}$, $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ iff $\vec{a}$ and $\vec{b}$ are perpendicular vectors.

**14.** Show that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $3\hat{i} + 7\hat{j} + \hat{k}$ and $5\hat{i} + 6\hat{j} + 2\hat{k}$ form the sides of a right-angled triangle.

**15.** If two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 2, |\vec{b}| = 1$ and $\vec{a}\cdot\vec{b} = 1$, then find the value of $(3\vec{a} - 5\vec{b})\cdot(2\vec{a} + 7\vec{b})$.

**16.** Let $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$ be two vectors. Show that the vectors $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ are perpendicular to each other.

**17.** If θ is the angle between two vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$, then find sin θ.

**18.** Find $|2a \cdot (-b \times 3\vec{c})|$, where

$\vec{a} = \hat{i} - \hat{j} + 2\hat{k},\ \vec{b} = 3\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 3\hat{k}$.

**19.** If $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}, b = \hat{i} - 2\hat{j} + \hat{k}$ and $\vec{c} = -3\hat{i} + \hat{j} + 2\hat{k}$, then find $\vec{a} \cdot (\vec{b} \times \vec{c})$.

**20.** If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, find $(\vec{r} \times \hat{i}) \cdot (\vec{r} \times \hat{j}) + xy$.

## Short Answer Type Questions (SA-II)

**21.** The two vectors $\hat{j} + \hat{k}$ and $3\hat{i} - \hat{j} + 4\hat{k}$ represent the two sides *AB* and *AC*, respectively of a $\Delta ABC$. Find the length of the median through *A*.

**22.** Find a vector of magnitude 5 units and parallel to the resultant of the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$.

**23.** Let $\vec{a} = 4\hat{i} + 5\hat{j} - \hat{k}, \vec{b} = \hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} - \hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{c}$ and $\vec{b}$ and $\vec{d} \cdot \vec{a} = 21$.

**24.** The scalar product of the vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ and hence find the unit vector along $\vec{b} + \vec{c}$.

**25.** If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$, show that $\vec{a} - \vec{d}$ is parallel to $\vec{b} - \vec{c}$, where $\vec{a} \neq \vec{d}$ and $\vec{b} \neq \vec{c}$.

**26.** Dot product of a vector with vectors $\hat{i} - \hat{j} + \hat{k}$, $2\hat{i} + \hat{j} - 3\hat{k}$ and $\hat{i} + \hat{j} + \hat{k}$ are respectively 4, 0 and 2. Find the vector.

**27.** Find the values of λ for which the angle between the vectors $\vec{a} = 2\lambda^2\hat{i} + 4\lambda\hat{j} + \hat{k}$ and $\vec{b} = 7\hat{i} - 2\hat{j} + \lambda\hat{k}$ is obtuse.

**28.** Using vectors, find the area of the triangle with vertices *A*(1, 1, 2), *B*(2, 3, 5) and *C*(1, 5, 5).

**29.** If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors such that $|\vec{a}| = 3, |\vec{b}| = 4$ and $|\vec{c}| = 5$ and each one of them is perpendicular to the sum of the other two, then find $|\vec{a} + \vec{b} + \vec{c}|$.

**30.** If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ represent two adjacent sides of a parallelogram, find unit vectors parallel to the diagonals of the parallelogram.

**31.** Using vectors, find the area of the triangle *ABC* with vertices *A*(1, 2, 3), *B*(2, –1, 4) and *C*(4, 5, –1)

**32.** If $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}, \vec{b} = 2\hat{i} + \hat{j}$ and $\vec{c} = 3\hat{i} - 4\hat{j} - 5\hat{k}$, then find a unit vector perpendicular to both of the vectors $(\vec{a} - \vec{b})$ and $(\vec{c} - \vec{b})$.]

**33.** Find a unit vector perpendicular to both of the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ where $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$.

**34.** If the sum of two unit vectors is a unit vector, prove that the magnitude of their difference is $\sqrt{3}$.

**35.** If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, find a vector $\vec{c}$, such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$.

## Long Answer Type Questions (LA)

**36.** If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to $\vec{a}, \vec{b}$ and $\vec{c}$. Also, find the angle which $\vec{a} + \vec{b} + \vec{c}$ makes with $\vec{a}$ or $\vec{b}$ or $\vec{c}$.

**37.** Show that the points *A*, *B*, *C* with position vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ respectively, are the vertices of a right-angled triangle. Hence find the area of the triangle.

**38.** The two adjacent sides of a parallelogram are $2\hat{i} - 4\hat{j} - 5\hat{k}$ and $2\hat{i} + 2\hat{j} + 3\hat{k}$. Find the two unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram.

**39.** If $\vec{a} = 3\hat{i} - \hat{j}$ and $\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$ then express $\vec{b}$ in the form $\vec{b} = \vec{b}_1 + \vec{b}_2$ where $\vec{b}_1 \parallel \vec{a}$ and $\vec{b}_2 \perp \vec{a}$.

**40.** If $\hat{i} + \hat{j} + \hat{k}, 2\hat{i} + 5\hat{j}, 3\hat{i} + 2\hat{j} - 3\hat{k}$ and $\hat{i} - 6\hat{j} - \hat{k}$ respectively are the position vectors of points *A*, *B*, *C* and *D*, then find the angle between the straight lines *AB* and *CD*. Find whether $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear or not.

# ANSWERS

## OBJECTIVE TYPE QUESTIONS

**1.** **(a) :** The given vectors are

$\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$, $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$

$\therefore$ Required sum $= \vec{a} + \vec{b} + \vec{c}$

$= (\hat{i} - 2\hat{j} + \hat{k}) + (-2\hat{i} + 4\hat{j} + 5\hat{k}) + (\hat{i} - 6\hat{j} - 7\hat{k})$

$= -4\hat{j} - \hat{k}$.

**2.** **(b) :** Here, $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$

$\therefore$ Its magnitude $= |\vec{a}|$

$= \sqrt{3^2 + (-2)^2 + 6^2} = \sqrt{9 + 4 + 36} = \sqrt{49} = 7.$

**3.** **(d) :** Let $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k} \Rightarrow (\vec{a}\cdot\hat{i})^2 = x^2$

Similarly, $(\vec{a}\cdot\hat{j})^2 = y^2$ and $(\vec{a}\cdot\hat{k})^2 = z^2$

$\therefore (\vec{a}\cdot\hat{i})^2 + (\vec{a}\cdot\hat{j})^2 + (\vec{a}\cdot\hat{k})^2 = x^2 + y^2 + z^2 = |\vec{a}|^2$

**4.** **(a) :** $\overrightarrow{EA} + \overrightarrow{EB} + \overrightarrow{EC} + \overrightarrow{ED}$

$= \overrightarrow{EA} + \overrightarrow{EB} - \overrightarrow{EA} - \overrightarrow{EB}$

[As diagonals of a rhombus bisect each other]

$= \vec{0}$

**5.** **(c) :** We have, $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 4\hat{i} + 4\hat{j} - 2\hat{k}$

Now, $\vec{a}\cdot\vec{b} = (2\hat{i} - \hat{j} + 2\hat{k})\cdot(4\hat{i} + 4\hat{j} - 2\hat{k})$

$= 8 - 4 - 4 = 0$. Therefore, $\vec{a}\cdot\vec{b} = 0 \Rightarrow \cos\theta = 0$

So, angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{2}$.

**6.** **(a) :** We have, $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$

$\therefore \vec{a}\cdot\vec{b} = (2\hat{i} + 3\hat{j} + 2\hat{k})\cdot(\hat{i} + 2\hat{j} + \hat{k}) = 2 + 6 + 2 = 10$

and $|\vec{b}| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6}$

Hence, projection of $\vec{a}$ on $\vec{b}$ is $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = \frac{10}{\sqrt{6}}$.

**7.** **(c) :** Let $\vec{a} = -3\hat{i} + 4\hat{j} - 8\hat{k}$, $\vec{b} = 5\hat{i} - 6\hat{j} + 4\hat{k}$

Let $C(\vec{c})$ be the point in $yz$-plane which divides $\overline{AB}$ in the ratio $r : 1$.

Then, $0 = \frac{5r - 3}{r + 1}$ $\quad$ ($\because$ In $yz$-plane, $x = 0$)

$\Rightarrow 5r - 3 = 0 \Rightarrow r = \frac{3}{5}$

Thus required ratio is 3 : 5

**8.** **(c) :** Let $\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}$

$\therefore |\vec{a}| = \sqrt{1 + 4 + 4} = \sqrt{9} = 3$

$\therefore$ Required vector $= \frac{9(\hat{i} - 2\hat{j} + 2\hat{k})}{3} = 3(\hat{i} - 2\hat{j} + 2\hat{k})$

**9.** **(b) :** We have, $\cos\frac{\pi}{3} = \frac{(\hat{i} + \hat{k})\cdot(\hat{i} + \hat{j} + a\hat{k})}{\sqrt{2}\sqrt{1 + 1 + a^2}}$

$\Rightarrow \frac{1}{2} = \frac{1 + a}{\sqrt{2}\sqrt{2 + a^2}} \Rightarrow \frac{1}{4} = \frac{(1 + a)^2}{2(2 + a^2)}$

$\Rightarrow 2 + a^2 = 2(1 + a^2 + 2a) \Rightarrow a^2 + 4a = 0 \Rightarrow a = 0, -4$

**10.** **(a) :** Given, $|\vec{a} - \vec{b}| = |\vec{a}| = |\vec{b}| = 1$

$\Rightarrow |\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}\cdot\vec{b} \Rightarrow 1 = 1 + 1 - 2|\vec{a}||\vec{b}|\cos\theta$

(Here $\theta$ is angle between $\vec{a}$ and $\vec{b}$)

$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

**11.** **(b) :** Here, $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$, b $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$

Now, $\vec{b}\times\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 1 \\ 3 & 1 & 2 \end{vmatrix} = 3\hat{i} + 5\hat{j} - 7\hat{k}$

$\therefore \vec{a}\cdot(\vec{b}\times\vec{c}) = (2\hat{i} + \hat{j} + 3\hat{k})\cdot(3\hat{i} + 5\hat{j} - 7\hat{k})$

$= 2\times3 + 1\times5 + 3\times(-7)$

$= 6 + 5 - 21 = -10$

**12.** **(b) :** Given, $|\vec{a}| = |\vec{b}|$, $\theta = 60°$ and $\vec{a}\cdot\vec{b} = \frac{9}{2}$

Now, $\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

$\Rightarrow \cos 60° = \frac{9/2}{|\vec{a}|^2} \Rightarrow \frac{1}{2} = \frac{9/2}{|\vec{a}|^2}$

$\Rightarrow |\vec{a}|^2 = 9 \Rightarrow |\vec{a}| = 3 \therefore |\vec{a}| = |\vec{b}| = 3$

**13.** **(b) :** We have, $\vec{w} = x\vec{u} + y\vec{v}$

$\Rightarrow 4\hat{i} + 3\hat{j} = x(\hat{i} + 2\hat{j}) + y(-2\hat{i} + \hat{j})$

$\Rightarrow (x - 2y - 4)\hat{i} + (2x + y - 3)\hat{j} = \vec{0}$

$\Rightarrow x - 2y - 4 = 0$ and $2x + y - 3 = 0$

$\Rightarrow x = 2$ and $y = -1$

**14.** **(b) :** We have, $\vec{a} = -2\hat{i} + \hat{j} - 5\hat{k}$

Direction cosines of the given vector are

$$\left(\frac{-2}{\sqrt{(-2)^2 + (1)^2 + (-5)^2}}, \frac{1}{\sqrt{(-2)^2 + (1)^2 + (-5)^2}}, \frac{-5}{\sqrt{(-2)^2 + (1)^2 + (-5)^2}}\right)$$

$$= \left(\frac{-2}{\sqrt{4 + 1 + 25}}, \frac{1}{\sqrt{4 + 1 + 25}}, \frac{-5}{\sqrt{4 + 1 + 25}}\right)$$

$\therefore$ Direction cosines are $\left(\frac{-2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{-5}{\sqrt{30}}\right)$

**15. (d) :** We have, $\vec{c} = (x-2)\,\vec{a} + \vec{b}$, $\vec{d} = (2x+1)\,\vec{a} - \vec{b}$ are collinear, then $\vec{c} = m\vec{d}$

$\Rightarrow \quad (x-2)\vec{a} + \vec{b} = m\left((2x+1)\vec{a} - \vec{b}\right)$

$\Rightarrow \quad -m = 1 \Rightarrow m = -1$

and $m\,(2x + 1) = x - 2 \Rightarrow -2x - 1 = x - 2 \Rightarrow x = \dfrac{1}{3}$

**16. (a) :** We have, $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 3 & 5 & -2 \end{vmatrix}$

$= \hat{i}(-2-15) - (-4-9)\,\hat{j} + (10-3)\,\hat{k} = -17\hat{i} + 13\hat{j} + 7\hat{k}$

Hence, $|\vec{a} \times \vec{b}| = \sqrt{(-17)^2 + (13)^2 + (7)^2} = \sqrt{507}$

**17. (c) :** $(\hat{i}+\hat{j}) \times (\hat{j}+\hat{k}) \cdot (\hat{k}+\hat{i}) = (\hat{i}\times\hat{j} + \hat{i}\times\hat{k} + \hat{j}\times\hat{k}) \cdot (\hat{k}+\hat{i})$

$= (\hat{k} - \hat{j} + \hat{i}) \cdot (\hat{k}+\hat{i}) = \hat{k}\cdot\hat{k} + \hat{i}\cdot\hat{i}$ $\quad (\because \hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0)$

$= |\hat{k}|^2 + |\hat{i}|^2 = 1 + 1 = 2$

**18. (b) :** Clearly, angle between $\vec{a}$ and $\vec{b}$ is $\dfrac{\pi}{2}$.

$\Rightarrow \quad \vec{a}\cdot\vec{b} = 0$

$\therefore \quad |\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b} = 1 + 1 + 0 = 2$

$\Rightarrow \quad |\vec{a}+\vec{b}| = \sqrt{2}$

**19. (c) :** Let $\vec{b} = \hat{i}$ and $\vec{c} = \hat{j}$ and $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

Now, $\vec{a}\cdot\vec{b} = a_1$, $\vec{a}\cdot\vec{c} = a_2$ and $\vec{a}\cdot\dfrac{\vec{b}\times\vec{c}}{|\vec{b}\times\vec{c}|} = \vec{a}\cdot\hat{k} = a_3$

$\therefore \quad (\vec{a}\cdot\vec{b})\vec{b} + (\vec{a}\cdot\vec{c})\vec{c} + \dfrac{\vec{a}\cdot(\vec{b}\times\vec{c})}{|\vec{b}\times\vec{c}|}(\vec{b}\times\vec{c})$

$= a_1\vec{b} + a_2\vec{c} + a_3\hat{k} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k} = \vec{a}$

**20. (c) :** $|\vec{a}+\vec{b}| < 1 \Rightarrow |\vec{a}+\vec{b}|^2 < 1$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b} < 1 \Rightarrow 1 + 1 + 2\vec{a}\cdot\vec{b} < 1$ $\quad [\because |\vec{a}| = |\vec{b}| = 1]$

$\Rightarrow \vec{a}\cdot\vec{b} < -\dfrac{1}{2} \Rightarrow |\vec{a}|\,|\vec{b}|\cos\theta < -\dfrac{1}{2}$

$\Rightarrow 1\times1\times\cos\theta < -\dfrac{1}{2} \Rightarrow \cos\theta < -\dfrac{1}{2}$

$\Rightarrow -1 \le \cos\theta < -\dfrac{1}{2} \Rightarrow \pi \ge \theta > \dfrac{2\pi}{3}$

**21. (d) :** $|\vec{a}| = 10$, $|\vec{b}| = 2$, $\vec{a}\cdot\vec{b} = 12$

We know, $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$\Rightarrow \quad 12 = 10 \times 2\cos\theta \quad \Rightarrow \quad \cos\theta = \dfrac{3}{5}$

$\therefore \quad \sin\theta = \dfrac{4}{5}$

Now, $|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta = 10 \times 2 \times \dfrac{4}{5} = 16$

**22. (b) :** $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = \vec{a}\cdot\vec{a} - \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{a} - \vec{b}\cdot\vec{b}$

$= \vec{a}\cdot\vec{a} - \vec{b}\cdot\vec{b} \quad [\because \vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}]$

$= |\vec{a}|^2 - |\vec{b}|^2 = [1^2 + 2^2 + (-3)^2] - [3^2 + (-1)^2 + 2^2] = 0$

$\Rightarrow \quad (\vec{a}+\vec{b}) \perp (\vec{a}-\vec{b})$

**23. (a) :** Let $\vec{a} = 2\hat{i} - 3\hat{k}$ and $\vec{b} = 4\hat{j} + 2\hat{k}$

The area of a parallelogram with $\vec{a}$ and $\vec{b}$ as its adjacent sides is given by $|\vec{a}\times\vec{b}|$.

Now, $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & -3 \\ 0 & 4 & 2 \end{vmatrix} = 12\hat{i} - 4\hat{j} + 8\hat{k}$

$\therefore \quad |\vec{a}\times\vec{b}| = \sqrt{(12)^2 + (-4)^2 + (8)^2} = \sqrt{144 + 16 + 64}$

$= \sqrt{224} = 4\sqrt{14}$ sq. units.

**24. (b) :** We have, $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$

$\therefore \quad 3\vec{a} + 2\vec{b} = 3(\hat{i} + \hat{j} - 2\hat{k}) + 2(2\hat{i} - 4\hat{j} + 5\hat{k})$

$= (3\hat{i} + 3\hat{j} - 6\hat{k}) + (4\hat{i} - 8\hat{j} + 10\hat{k}) = 7\hat{i} - 5\hat{j} + 4\hat{k}$

$\therefore$ The direction ratios of the vector $3\vec{a} + 2\vec{b}$ are 7, – 5, 4.

**25. (b) :** We have $\vec{a}\cdot\vec{b} = 2\sqrt{3}$, $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 4$

Let $\theta$ be the angle between $\vec{a}$ and $\vec{b}$

$\therefore \quad \vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$\Rightarrow \quad 2\sqrt{3} = \sqrt{3}\cdot 4\cdot\cos\theta \quad \Rightarrow \quad \cos\theta = \dfrac{1}{2} \quad \Rightarrow \quad \theta = \dfrac{\pi}{3}$

**26. (d) :** Given points are $2\vec{a} - 3\vec{b}$ and $\vec{a} + \vec{b}$ and Given ratio = 3 : 1

$\therefore \quad$ Required vector $= \dfrac{(2\vec{a} - 3\vec{b})\times 1 + (\vec{a} + \vec{b})\times 3}{3+1}$

$= \dfrac{2\vec{a} - 3\vec{b} + 3\vec{a} + 3\vec{b}}{4} = \dfrac{5}{4}\vec{a}$

**27. (c) :** We have, $-3 \le \lambda \le 3 \Rightarrow |\lambda| \le 3$

Now, $|\lambda||\vec{a}| \le 3|\vec{a}| \quad \Rightarrow \quad |\lambda\vec{a}| \le 12$

$\therefore \quad$ Range of $|\lambda\vec{a}|$ is [0, 12]

**28. (b) :** Given, $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 27 \\ 1 & p & q \end{vmatrix} = 0$

$\Rightarrow \quad \hat{i}\,(6q - 27p) - \hat{j}\,(2q - 27) + \hat{k}\,(2p - 6) = 0$

$\Rightarrow \quad 6q - 27p = 0,\ 2q - 27 = 0$ and $2p - 6 = 0$

$\Rightarrow \quad q = \dfrac{27}{2}$ and $p = 3$.

**29. (c) :** We have $\vec{a}, \vec{b}, \vec{c}$ are unit vectors.

Therefore, $|\vec{a}| = 1, |\vec{b}| = 1$ and $|\vec{c}| = 1$

Also, $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ (given)

$\Rightarrow \quad |\vec{a}+\vec{b}+\vec{c}|^2 = 0$

$\Rightarrow \quad |\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=0$

$\Rightarrow \quad 1+1+1+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=0$

$\Rightarrow \quad 3+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=0$

$\Rightarrow \quad (\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=-\frac{3}{2}$

**30. (a) :** $\vec{a}=3\hat{i}-6\hat{j}+\hat{k}$ and $\vec{b}=2\hat{i}-4\hat{j}+\lambda\hat{k}$

Since, $\vec{a}$ and $\vec{b}$ are parallel $\quad \therefore \quad \vec{a}\times\vec{b}=\vec{0}$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -6 & 1 \\ 2 & -4 & \lambda \end{vmatrix} = \vec{0}$$

$\Rightarrow \quad (-6\lambda+4)\hat{i}-(3\lambda-2)\hat{j}+(-12+12)\hat{k}=\vec{0}$

$\Rightarrow \quad (-6\lambda+4)\hat{i}+(2-3\lambda)\hat{j}=0\hat{i}+0\hat{j}$

Comparing coefficients of $\hat{i}$ and $\hat{j}$, we get

$-6\lambda+4=0$ and $2-3\lambda=0 \Rightarrow \lambda=2/3$

**31. (b) :** The given vectors will be at right angles if their dot product vanishes, *i.e.*,

$(2\hat{i}-4\hat{j}+\hat{k})\cdot(4\hat{i}-8\hat{j}+\lambda\hat{k})=0$

$\Rightarrow \quad 8+32+\lambda=0 \Rightarrow \lambda=-40$

**32. (a) :** Let $\vec{a}=\hat{i}+3\hat{j}+7\hat{k}$ and $\vec{b}=2\hat{i}-3\hat{j}+6\hat{k}$.

Now, projection of $\vec{a}$ on $\vec{b}=\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$$=\frac{(\hat{i}+3\hat{j}+7\hat{k})\cdot(2\hat{i}-3\hat{j}+6\hat{k})}{\sqrt{2^2+(-3)^2+6^2}}=\frac{2-9+42}{\sqrt{4+9+36}}=\frac{35}{7}=5$$

**33. (c) :** Let $A(2, 5, 0)$ and $B(-3, 7, 4)$

$\therefore \quad$ Required vector $=(-3-2)\hat{i}+(7-5)\hat{j}+(4-0)\hat{k}$

$=-5\hat{i}+2\hat{j}+4\hat{k}$

**34. (d) :** $\vec{a}=2\hat{i}-3\hat{j}+2\hat{k}$ and $\vec{b}=2\hat{i}+3\hat{j}+\hat{k}$

$$\vec{a}\times\vec{b}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix} = -9\hat{i}+2\hat{j}+12\hat{k}$$

Area of $\Delta OAB=\frac{1}{2}|\vec{a}\times\vec{b}|=\frac{1}{2}\sqrt{81+4+144}=\frac{1}{2}\sqrt{229}$

**35. (b) :** Given that, $|\vec{a}|=3, |\vec{b}|=4$ and $\vec{a}+\lambda\vec{b}$ is perpendicular to $\vec{a}-\lambda\vec{b}$.

$\therefore \quad (\vec{a}+\lambda\vec{b})\cdot(\vec{a}-\lambda\vec{b})=0$

$\Rightarrow \quad \vec{a}\cdot\vec{a}-\vec{a}\cdot\vec{b}\lambda+\lambda\vec{b}\cdot\vec{a}-\lambda^2\vec{b}\cdot\vec{b}=0$

$\Rightarrow \quad |\vec{a}|^2-\lambda^2|\vec{b}|^2=0 \Rightarrow \lambda^2=\frac{|\vec{a}|^2}{|\vec{b}|^2} \Rightarrow \lambda=\frac{|\vec{a}|}{|\vec{b}|}=\frac{3}{4}$

**36. (c) :** Position vector of $\overrightarrow{AB}$

$=(2-1)\hat{i}+(1-1)\hat{j}+(3-1)\hat{k}=\hat{i}+2\hat{k}$

**37 (b) :** Position vector of $\overrightarrow{AC}$

$=(3-1)\hat{i}+(2-1)\hat{j}+(2-1)\hat{k}=2\hat{i}+\hat{j}+\hat{k}$

**38. (d) :** Position vector of $\overrightarrow{AD}$

$=(3-1)\hat{i}+(3-1)\hat{j}+(4-1)\hat{k}=2\hat{i}+2\hat{j}+3\hat{k}$

**39. (b) :** Area of $\Delta ABC=\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|$

$$\text{Now, } \overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 2 \\ 2 & 1 & 1 \end{vmatrix}=\hat{i}(0-2)-\hat{j}(1-4)+\hat{k}(1-0)$$

$=-2\hat{i}+3\hat{j}+\hat{k}$

$\Rightarrow \quad |\overrightarrow{AB}\times\overrightarrow{AC}|=\sqrt{(-2)^2+3^2+1^2}$

$=\sqrt{4+9+1}=\sqrt{14}$

$\therefore \quad$ Area of $\Delta ABC=\frac{1}{2}\sqrt{14}$ sq. units

**40. (a) :** Unit vector along $\overrightarrow{AD}=\frac{\overrightarrow{AD}}{|\overrightarrow{AD}|}$

$$=\frac{2\hat{i}+2\hat{j}+3k}{\sqrt{2^2+2^2+3^2}}=\frac{2\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{4+4+9}}=\frac{1}{\sqrt{17}}(2\hat{i}+2\hat{j}+3\hat{k})$$

**41. (a) :** $\vec{a}=\hat{i}+4\hat{j}+2\hat{k}, \vec{b}=3\hat{i}-3\hat{j}-2\hat{k}$

and $\vec{c}=-2\hat{i}+2\hat{j}+6\hat{k}$

$\therefore \quad \vec{a}+\vec{b}+\vec{c}=2\hat{i}+3\hat{j}+6\hat{k}$

**42. (c) :** Using triangle law of addition in $\Delta ABC$, we get

$\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=\vec{0}$, which can be rewritten as

$\overrightarrow{AB}+\overrightarrow{BC}-\overrightarrow{AC}=\vec{0}$ or $\overrightarrow{AB}-\overrightarrow{CB}+\overrightarrow{CA}=\vec{0}$

**43. (c) :** We have, $A(1,4, 2)$, $B(3, -3, -2)$ and $C(-2, 2, 6)$

Now, $\overrightarrow{AB}=\vec{b}-\vec{a}=2\hat{i}-7\hat{j}-4\hat{k}$

and $\overrightarrow{AC}=\vec{c}-\vec{a}=-3\hat{i}-2\hat{j}+4\hat{k}$

$$\therefore \quad \overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -7 & -4 \\ -3 & -2 & 4 \end{vmatrix}$$

$=\hat{i}(-28-8)-\hat{j}(8-12)+\hat{k}(-4-21)=-36\hat{i}+4\hat{j}-25\hat{k}$

Now, $|\overrightarrow{AB}\times\overrightarrow{AC}|=\sqrt{(-36)^2+4^2+(-25)^2}$

$=\sqrt{1296+16+625}=\sqrt{1937}$

$\therefore \quad$ Area of $\Delta ABC=\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|=\frac{1}{2}\sqrt{1937}$ sq. units

**44. (d) :** If the given points lie on the straight line, then the points will be collinear and so area of $\Delta ABC = 0$.

$\Rightarrow \quad |\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}| = 0$

[$\because$ If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the three vertices $A$, $B$ and $C$ of $\Delta ABC$, then area of triangle $= \frac{1}{2}|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$]

**45. (b) :** Here, $|\vec{a}| = \sqrt{2^2 + 3^2 + 6^2} = \sqrt{4 + 9 + 36}$

$= \sqrt{49} = 7$

$\therefore$ Unit vector in the direction of vector $\vec{a}$ is

$$\hat{a} = \frac{2\hat{i} + 3\hat{j} + 6\hat{k}}{7} = \frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$$

**46. (b) :** Here, (4, 10) are the coordinates of $A$.

$\therefore$ P.V. of $A = 4\hat{i} + 10\hat{j}$

**47. (c) :** Here, (9, 7) are the coordinates of $B$.

$\therefore$ P.V. of $B = 9\hat{i} + 7\hat{j}$

**48. (b) :** Here, P.V. of $A = 4\hat{i} + 10\hat{j}$ and P.V. of $C = 4\hat{i} + 2\hat{j}$

$\therefore \quad \overline{AC} = (4 - 4)\ \hat{i} + (2 - 10)\ \hat{j} = -8\ \hat{j}$

**49. (a) :** Here $\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\therefore \quad |\vec{A}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{1 + 4 + 9} = \sqrt{14}$

$\therefore \quad \hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{14}} = \frac{1}{\sqrt{14}}\hat{i} + \frac{2}{\sqrt{14}}\hat{j} + \frac{3}{\sqrt{14}}\hat{k}$

**50. (d) :** We have, $\vec{A} = 4\hat{i} + 3\hat{j}$ and $\vec{B} = 3\hat{i} + 4\hat{j}$

$\therefore \quad |\vec{A}| = \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25} = 5$

and $|\vec{B}| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

Thus, $|\vec{A}| + |\vec{B}| = 5 + 5 = 10$

**51. (b) :** $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$

Resultant of $\vec{a}$ and $\vec{b}$ is $\vec{a} + \vec{b}$

$= (2\hat{i} + \hat{j} + \hat{k}) + (\hat{i} + 2\hat{j} + 3\hat{k}) = 3\hat{i} + 3\hat{j} + 4\hat{k}$

$\therefore \quad |\vec{a} + \vec{b}| = \sqrt{3^2 + 3^2 + 4^2} = \sqrt{9 + 9 + 16} = \sqrt{34}$

Also, the magnitude of a vector can never be negative. Hence, both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.

**52. (d) :** Sum of the given vectors

$= (\hat{i} + \hat{j} + \hat{k}) + (2\hat{i} - \hat{j} - \hat{k}) + (2\hat{j} + 6\hat{k}) = 3\hat{i} + 2\hat{j} + 6\hat{k}$

$\therefore$ The unit vector in the direction of the sum of the given vectors

$$= \frac{3\hat{i} + 2\hat{j} + 6\hat{k}}{\sqrt{3^2 + 2^2 + 6^2}} = \frac{3\hat{i} + 2\hat{j} + 6\hat{k}}{\sqrt{9 + 4 + 36}} = \frac{1}{7}(3\hat{i} + 2\hat{j} + 6\hat{k})$$

Hence, Assertion is wrong.

Also, $\frac{\vec{a}}{|\vec{a}|}$ is a unit vector which is parallel to $\vec{a}$.

Hence, Reason is correct.

**53. (a) :** $\vec{a} = \hat{i} + \hat{j} - 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$

$\vec{a} \cdot \vec{b} = (\hat{i} + \hat{j} - 3\hat{k}) \cdot (2\hat{i} + j + \hat{k})$

$= 1 \cdot 2 + 1 \cdot 1 + (-3) \cdot 1 = 2 + 1 - 3 = 0$

$\Rightarrow \quad \cos\theta = 0 \Rightarrow \quad \theta = \frac{\pi}{2}$

Hence, $\vec{a}$ and $\vec{b}$ are perpendicular to each other. Hence, both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

**54. (d) :** $\vec{a} = \hat{i} + 2\hat{j}$, $\vec{b} = 2\hat{i} + \hat{j}$

Diagonals of the parallelogram are along $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$.

Now, $\vec{a} + \vec{b} = (\hat{i} + 2\hat{j}) + (2\hat{i} + \hat{j}) = 3\hat{i} + 3\hat{j}$

and $\vec{a} - \vec{b} = (\hat{i} + 2\hat{j}) - (2\hat{i} + \hat{j}) = -\hat{i} + \hat{j}$

Let $\theta$ be the angle between these vectors, then

$$\cos\theta = \frac{(3\hat{i} + 3\hat{j}) \cdot (-\hat{i} + \hat{j})}{\sqrt{9 + 9}\sqrt{1 + 1}} = \frac{-3 + 3}{\sqrt{18}\sqrt{2}} = 0$$

$\Rightarrow \quad \theta = 90°$

Hence, Assertion is wrong and Reason is correct.

**55. (b) :** We have, $|\vec{a}| = 3$, $|\vec{b}| = 4$, $|\vec{c}| = 5$ and

$\vec{a} + \vec{b} + \vec{c} = \vec{0} \Rightarrow \quad (\vec{a} + \vec{b} + \vec{c})^2 = 0$

$\Rightarrow \quad |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$

$\Rightarrow \quad (3)^2 + (4)^2 + (5)^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$

$\Rightarrow \quad \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -\frac{1}{2}[9 + 16 + 25] = -\frac{1}{2}(50) = -25$

Now, $\vec{a} + \vec{b} + \vec{c} = \vec{0} \quad \Rightarrow \quad \vec{b} + \vec{c} = -\vec{a}$

$\Rightarrow \quad |\vec{b} + \vec{c}|^2 = |-\vec{a}|^2 \quad \Rightarrow \quad \vec{b}^2 + \vec{c}^2 + 2\vec{b} \cdot \vec{c} = \vec{a}^2$

$\Rightarrow \quad \vec{b}^2 + \vec{c}^2 + 2\vec{b}\vec{c}\cos\theta = \vec{a}^2$

$\Rightarrow \quad \cos\theta = \frac{\vec{a}^2 - \vec{b}^2 - \vec{c}^2}{2\vec{b}\vec{c}}$

Hence, both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.

**56. (a) :** Required length $= \left|\frac{(3\hat{i} - \hat{j} - 2\hat{k}) \cdot (\hat{i} + 2\hat{j} - 3\hat{k})}{\sqrt{1^2 + 2^2 + (-3)^2}}\right|$

$\left|\frac{3 - 2 + 6}{\sqrt{1 + 4 + 9}}\right| = \frac{7}{\sqrt{14}}$

Also, vector projection of $\vec{a}$ on $\vec{b} = (\vec{a} \cdot \hat{b}) = \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}\right)$

Hence, both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

**57.** **(d) :** $(\vec{a}\times\vec{b})^2+(\vec{a}\cdot\vec{b})^2=|\vec{a}\times\vec{b}|^2+(\vec{a}\cdot\vec{b})^2$

$$=(\vec{a}\vec{b}\sin\theta)^2+(\vec{a}\vec{b}\cos\theta)^2=\vec{a}^2\vec{b}^2$$

Hence, Assertion is wrong.
But $\sin^2\theta+\cos^2\theta=1$
Hence, Reason is correct.

**58.** **(d) :** $(\vec{a}\times\vec{b})^2+(\vec{a}\cdot\vec{b})^2=400, |\vec{a}|=4$
We know that,

$(\vec{a}\times\vec{b})^2+(\vec{a}\cdot\vec{b})^2=|\vec{a}|^2|\vec{b}|^2$

$\Rightarrow\ 400=(4)^2|\vec{b}|^2\Rightarrow 16|\vec{b}|^2=400$

$\Rightarrow\ |\vec{b}|^2=25\Rightarrow|\vec{b}|=5$

Hence, Assertion is wrong.

$(\vec{a}\times\vec{b})^2+(\vec{a}\cdot\vec{b})^2=|\vec{a}\times\vec{b}|^2+(\vec{a}\cdot\vec{b})^2$

$=(\vec{a}\vec{b}\sin\theta)^2+(\vec{a}\vec{b}\cos\theta)^2=\vec{a}^2\vec{b}^2$

$\Rightarrow\ (\vec{a}\times\vec{b})^2=\vec{a}^2\vec{b}^2-(\vec{a}\cdot\vec{b})^2$

Hence, Reason is correct.

**59.** **(a) :** Projection of $\vec{a}$ on $\vec{b}=\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$$=\frac{(2\hat{i}+3\hat{j}-\hat{k})\cdot(-\hat{i}+3\hat{j}+4\hat{k})}{\sqrt{(-1)^2+(3)^2+(4)^2}}=\frac{-2+9-4}{\sqrt{26}}=\frac{3}{\sqrt{26}}$$

$\therefore$ Assertion and Reason are correct and Reason is the correct explanation of Assertion.

**60.** **(b) :** If $A$, $B$, $C$ are collinear, then $\overline{AB}=k\overline{AC}$

$\therefore\ \overline{AB}\times\overline{AC}=\vec{0}\ \Rightarrow\ (\vec{b}-\vec{a})\times(\vec{c}-\vec{a})=0$

$\Rightarrow\ \vec{b}\times\vec{c}+\vec{a}\times\vec{b}+\vec{c}\times\vec{a}=\vec{0}$ *i.e.*, $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=\vec{0}$

Hence, both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.

## SUBJECTIVE TYPE QUESTIONS

**1.** We have, $l=\cos\frac{\pi}{3}=\frac{1}{2}, m=\cos\frac{\pi}{4}=\frac{1}{\sqrt{2}}$ and $n=\cos\theta$

Now, $l^2+m^2+n^2=1$

$\Rightarrow\ \left(\frac{1}{2}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2+n^2=1$

$\Rightarrow\ \frac{1}{4}+\frac{1}{2}+n^2=1\Rightarrow n^2=\frac{1}{4}\ \Rightarrow\ n=\pm\frac{1}{2}$

$\Rightarrow\ \cos\theta=\pm\frac{1}{2}$

But $\theta$ is an acute angle (given).

$\therefore\ \theta=\cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}$

**2.** Required sum $=\vec{a}+\vec{b}+\vec{c}$

$=(\hat{i}-3\hat{k})+(2\hat{j}-\hat{k})+(2\hat{i}-3\hat{j}+2\hat{k})$

$=3\hat{i}-\hat{j}-2\hat{k}.$

**3.** Let $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=4\hat{i}-3\hat{j}+2\hat{k}$. Then, the sum of the given vectors is

$\vec{c}=\vec{a}+\vec{b}=(2+4)\hat{i}+(3-3)\hat{j}+(-1+2)\hat{k}=6\hat{i}+\hat{k}$

and $|\vec{c}|=|\vec{a}+\vec{b}|=\sqrt{6^2+1^2}=\sqrt{36+1}=\sqrt{37}$

$\therefore$ Unit vector, $\hat{c}=\dfrac{\vec{c}}{|\vec{c}|}=\dfrac{6\hat{i}+\hat{k}}{\sqrt{37}}=\dfrac{6}{\sqrt{37}}\hat{i}+\dfrac{1}{\sqrt{37}}\hat{k}$

**4.** Required position vector

$=\dfrac{2(\vec{a}+2\vec{b})-1(2\vec{a}-\vec{b})}{2-1}=5\vec{b}$

**5.** $|\vec{a}\times\vec{b}|^2+|\vec{a}\cdot\vec{b}|^2$

$=\{|\vec{a}||\vec{b}|\sin\theta\}^2+\{|\vec{a}||\vec{b}|\cos\theta\}^2$

$=|\vec{a}|^2|\vec{b}|^2\sin^2\theta+|\vec{a}|^2|\vec{b}|^2\cos^2\theta$

$=|\vec{a}|^2|\vec{b}|^2=25\times16$ $\quad[\because\ |\vec{a}|=5$ and $|\vec{b}|=4]$

$=400$

**6.** Let $\vec{a}=\hat{i}-3\hat{k}$ and $\vec{b}=2\hat{j}+\hat{k}$

The area of a parallelogram with $\vec{a}$ and $\vec{b}$ as its adjacent sides is given by $|\vec{a}\times\vec{b}|$.

Now, $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&0&-3\\0&2&1\end{vmatrix}=6\hat{i}-\hat{j}+\hat{k}$

$\therefore\ |\vec{a}\times\vec{b}|=\sqrt{(6)^2+(-1)^2+(2)^2}=\sqrt{36+1+4}$

$=\sqrt{41}$ sq. units.

**7.** Projection of $\vec{a}$ on $\vec{b}=\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$$=\frac{(2\hat{i}+3\hat{j}+2\hat{k})\cdot(2\hat{i}+2\hat{j}+\hat{k})}{\sqrt{(2)^2+(2)^2+(1)^2}}=\frac{4+6+2}{3}=\frac{12}{3}=4$$

**8.** Let $\theta$ be the angle between the unit vectors $\vec{a}$ and $\vec{b}$.

$\therefore\ \cos\theta=\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}=\vec{a}\cdot\vec{b}$ $\quad(\because\ |\vec{a}|=1=|\vec{b}|)$ ...(1)

Now, $1=|\sqrt{2}\vec{a}-\vec{b}|$

$\Rightarrow\ 1=|\sqrt{2}\vec{a}-\vec{b}|^2=(\sqrt{2}\vec{a}-\vec{b})\cdot(\sqrt{2}\vec{a}-\vec{b})$

$=2|\vec{a}|^2-\sqrt{2}\vec{a}\cdot\vec{b}-\vec{b}\cdot\sqrt{2}\vec{a}+|\vec{b}|^2=2-2\sqrt{2}\vec{a}\cdot\vec{b}+1$

$(\because\ \vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a})$

$=3-2\sqrt{2}\ \vec{a}\cdot\vec{b}$

$\Rightarrow\ \vec{a}\cdot\vec{b}=\dfrac{1}{\sqrt{2}}\Rightarrow\ \cos\theta=\dfrac{1}{\sqrt{2}}$ $\quad$[By using (1)]

$\therefore\ \theta=\pi/4$

**9.** Here, $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and vector along $x$-axis is $\hat{i}$.

$\therefore$ Angle between $\vec{a}$ and $\hat{i}$ is given by

$$\cos\theta = \frac{\vec{a}.\hat{i}}{\sqrt{1^2+1^2+1^2}.\sqrt{1^2}} = \frac{(\hat{i}+\hat{j}+\hat{k})\cdot\hat{i}}{\sqrt{3}\cdot 1} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

**10.** Here $|\vec{a}+\vec{b}| = |\vec{a}|$

$\Rightarrow |\vec{a}+\vec{b}|^2 = |\vec{a}|^2 \Rightarrow (\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) = \vec{a}\cdot\vec{a}$

$\Rightarrow \vec{a}\cdot\vec{a}+\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{a}+\vec{b}\cdot\vec{b} = \vec{a}\cdot\vec{a}$

$\Rightarrow 2\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{b} = 0 \qquad [\because \vec{b}\cdot\vec{a} = \vec{a}\cdot\vec{b}]$

$\Rightarrow (2\vec{a}+\vec{b})\cdot\vec{b} = 0 \Rightarrow (2\vec{a}+\vec{b}) \perp \vec{b}$

**11.** Position vector which divides the line segment joining points with position vectors $3\vec{a}+\vec{b}$ and $\vec{a}-3\vec{b}$ in the ratio 2 : 1 externally is given by

$$\frac{2(\vec{a}-3\vec{b})-1(3\vec{a}+\vec{b})}{2-1} = \frac{2\vec{a}-6\vec{b}-3\vec{a}-\vec{b}}{1}$$

$$= -\vec{a}-7\vec{b}$$

**12.** Here, $\vec{a} = 5\hat{i}+6\hat{j}-2\hat{k}$ and $\vec{b} = 7\hat{i}+6\hat{j}+2\hat{k}$

Vector perpendicular to both $\vec{a}$ and $\vec{b}$ is

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 6 & -2 \\ 7 & 6 & 2 \end{vmatrix}$$

$$= \hat{i}(12+12)-\hat{j}(10+14)+\hat{k}(30-42)$$

$$= 24\hat{i}-24\hat{j}-12\hat{k} = 12(2\hat{i}-2\hat{j}-\hat{k})$$

$\therefore$ Unit vector perpendicular to both $\vec{a}$ and $\vec{b}$

$$= \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|} = \frac{24\hat{i}-24\hat{j}-12\hat{k}}{\sqrt{576+576+144}} = \frac{12(2\hat{i}-2\hat{j}-\hat{k})}{\sqrt{1296}}$$

$$= \frac{12(2\hat{i}-2\hat{j}-\hat{k})}{36} = \frac{1}{3}(2\hat{i}-2\hat{j}-\hat{k})$$

**13.** For any two non-zero vectors $\vec{a}$ and $\vec{b}$, we have

$|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}| \Rightarrow |\vec{a}+\vec{b}|^2 = |\vec{a}-\vec{b}|^2$

$\Rightarrow \vec{a}^2+\vec{b}^2+2\vec{a}\cdot\vec{b} = \vec{a}^2+\vec{b}^2-2\vec{a}\cdot\vec{b}$

$\Rightarrow 4\vec{a}\cdot\vec{b} = 0 \Rightarrow \vec{a}\cdot\vec{b} = 0$

So, $\vec{a}$ and $\vec{b}$ are perpendicular vectors.

**14.** Let $A(2\hat{i}-\hat{j}+\hat{k})$, $B(3\hat{i}+7\hat{j}+\hat{k})$ and $C(5\hat{i}+6\hat{j}+2\hat{k})$

Then, $\overrightarrow{AB} = (3-2)\hat{i}+(7+1)\hat{j}+(1-1)\hat{k} = \hat{i}+8\hat{j}$

$\overrightarrow{AC} = (5-2)\hat{i}+(6+1)\hat{j}+(2-1)\hat{k} = 3\hat{i}+7\hat{j}+\hat{k}$

$\overrightarrow{BC} = (5-3)\hat{i}+(6-7)\hat{j}+(2-1)\hat{k} = 2\hat{i}-\hat{j}+\hat{k}$

Now, angle between $\overrightarrow{AC}$ and $\overrightarrow{BC}$ is given by

$$\Rightarrow \cos\theta = \frac{\overrightarrow{AC}\cdot\overrightarrow{BC}}{|\overrightarrow{AC}||\overrightarrow{BC}|} = \frac{6-7+1}{\sqrt{9+49+1}\sqrt{4+1+1}}$$

$\Rightarrow \cos\theta = 0 \Rightarrow AC \perp BC$

So, $A$, $B$, $C$ are the vertices of right angled triangle.

**15.** We have $|\vec{a}| = 2$, $|\vec{b}| = 1$ and $\vec{a}\cdot\vec{b} = 1$

Now, $(3\vec{a}-5\vec{b})\cdot(2\vec{a}+7\vec{b})$

$= 6|\vec{a}|^2 + 21\,\vec{a}\cdot\vec{b} - 10\vec{a}\cdot\vec{b} - 35|\vec{b}|^2$

$= 6|\vec{a}|^2 + 11\vec{a}\cdot\vec{b} - 35|\vec{b}|^2$

$= 6(2)^2 + 11(1) - 35(1)^2 = 24 + 11 - 35 = 0$

**16.** Given, $\vec{a} = \hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b} = 3\hat{i}-\hat{j}+2\hat{k}$

Now, $\vec{a}+\vec{b} = 4\hat{i}+\hat{j}-\hat{k}$

Also, $\vec{a}-\vec{b} = -2\hat{i}+3\hat{j}-5\hat{k}$

Now, $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = (4\hat{i}+\hat{j}-\hat{k})\cdot(-2\hat{i}+3\hat{j}-5\hat{k})$

$= (4)(-2) + (1)(3) + (-1)(-5) = -8 + 3 + 5 = 0$

Hence, $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ are perpendicular to each other.

**17.** Let $\vec{a} = \hat{i}-2\hat{j}+3\hat{k}$, $\vec{b} = 3\hat{i}-2\hat{j}+\hat{k}$

Now, $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$$\Rightarrow (\hat{i}-2\hat{j}+3\hat{k})\cdot(3\hat{i}-2\hat{j}+\hat{k}) = \sqrt{(1)^2+(-2)^2+(3)^2} \times \sqrt{(3)^2+(-2)^2+(1)^2}\cos\theta$$

$\Rightarrow 3+4+3 = \sqrt{14}\times\sqrt{14}\cos\theta$

$$\Rightarrow \cos\theta = \frac{10}{14} = \frac{5}{7}$$

$$\therefore \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{25}{49}} = \sqrt{\frac{24}{49}}$$

$$\Rightarrow \sin\theta = \frac{2\sqrt{6}}{7}$$

**18.** Given, $\vec{a} = \hat{i}-\hat{j}+2\hat{k}$, $\vec{b} = 3\hat{i}+4\hat{j}-5\hat{k}$ and $\vec{c} = 2\hat{i}-\hat{j}+3\hat{k}$

$\therefore \quad 2\vec{a} = 2\hat{i}-2\hat{j}+4\hat{k}$

$-\vec{b} = -3\hat{i}-4\hat{j}+5\hat{k}$

$3\vec{c} = 6\hat{i}-3\hat{j}+9\hat{k}$

$$\text{Now, } 2\vec{a}\cdot(-\vec{b}\times 3\vec{c}) = \begin{vmatrix} 2 & -2 & 4 \\ -3 & -4 & 5 \\ 6 & -3 & 9 \end{vmatrix}$$

$= 2(-36 + 15) + 2(-27 - 30) + 4(9 + 24)$

$= 2(-21) - 2(57) + 4(33)$

$= -42 - 114 + 132 = -24$

$\therefore \quad |2\vec{a}\cdot(-\vec{b}\times 3\vec{c})| = |-24| = 24$

**19.** Given, $\vec{a} = 2\hat{i}+3\hat{j}+\hat{k}$, $\vec{b} = \hat{i}-2\hat{j}+\hat{k}$ and $\vec{c} = -3\hat{i}+\hat{j}+2\hat{k}$

$$\text{Now, } \vec{b}\times\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ -3 & 1 & 2 \end{vmatrix}$$

$= \hat{i}(-4-1) - \hat{j}(2+3) + \hat{k}(1-6)$
$= -5\hat{i} - 5\hat{j} - 5\hat{k}$

$\therefore \quad \vec{a}\cdot(\vec{b}\times\vec{c}) = (2\hat{i}+3\hat{j}+\hat{k})\cdot(-5\hat{i}-5\hat{j}-5\hat{k})$
$= -10 - 15 - 5 = -30$

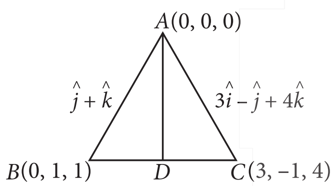
**20.** $(\vec{r}\times\hat{i})\cdot(\vec{r}\times\hat{j}) + xy$

$= [(x\hat{i}+y\hat{j}+z\hat{k})\times\hat{i}]\cdot[(x\hat{i}+y\hat{j}+z\hat{k})\times\hat{j})] + xy$

$= (-y\hat{k}+z\hat{j})\cdot(x\hat{k}-z\hat{i}) + xy = -xy + xy = 0$

**21.** Take $A$ to be as origin (0, 0, 0).

$\therefore$ Coordinates of $B$ are (0, 1, 1) and coordinates of $C$ are (3, –1, 4).




Let $D$ be the mid point of $BC$ and $AD$ is a median of $\Delta ABC$.

$\therefore$ Coordinates of $D$ are $\left(\frac{3}{2}, 0, \frac{5}{2}\right)$

So, length of $AD = \sqrt{\left(\frac{3}{2}-0\right)^2 + (0)^2 + \left(\frac{5}{2}-0\right)^2}$

$= \sqrt{\frac{9}{4}+\frac{25}{4}} = \frac{\sqrt{34}}{2}$ units

**22.** $\vec{a} = 2\hat{i}+3\hat{j}-\hat{k}, \ \vec{b} = \hat{i}-2\hat{j}+\hat{k}$

$\therefore \quad \vec{a}+\vec{b} = (2\hat{i}+3\hat{j}-\hat{k}) + (\hat{i}-2\hat{j}+\hat{k}) = 3\hat{i}+\hat{j}$

$|\vec{a}+\vec{b}| = \sqrt{3^2+1^2} = \sqrt{10}$

$\therefore$ A vector of magnitude 5 units in the direction of

$\vec{a}+\vec{b}$ is $\frac{5(\vec{a}+\vec{b})}{|\vec{a}+\vec{b}|} = \frac{5(3\hat{i}+\hat{j})}{\sqrt{10}}$

**23.** Let $\vec{d} = x\hat{i}+y\hat{j}+z\hat{k}$

Now, it is given that, $\vec{d}$ is perpendicular to

$\vec{b} = \hat{i}-4\hat{j}+5\hat{k}$ and $\vec{c} = 3\hat{i}+\hat{j}-\hat{k}$

$\therefore \quad \vec{d}\cdot\vec{b} = 0$ and $\vec{d}\cdot\vec{c} = 0$

$\Rightarrow \quad x - 4y + 5z = 0$ ...(i)

and $3x + y - z = 0$ ...(ii)

Also, $\vec{d}\cdot\vec{a} = 21$, where $\vec{a} = 4\hat{i}+5\hat{j}-\hat{k}$

$\Rightarrow \quad 4x + 5y - z = 21$ ...(iii)

Eliminating $z$ from (i) and (ii), we get

$16x + y = 0$ ...(iv)

Eliminating $z$ from (ii) and (iii), we get

$x + 4y = 21$ ...(v)

Solving (iv) and (v), we get

$x = \frac{-1}{3}, y = \frac{16}{3}$

Putting the values of $x$ and $y$ in (i), we get $z = \frac{13}{3}$

$\therefore \quad \vec{d} = \frac{-1}{3}\hat{i} + \frac{16}{3}\hat{j} + \frac{13}{3}\hat{k}$ is the required vector.

**24.** Here, $\vec{a} = \hat{i}+\hat{j}+\hat{k}$; $\vec{b} = 2\hat{i}+4\hat{j}-5\hat{k}$ and

$\vec{c} = \lambda\hat{i}+2\hat{j}+3\hat{k}$

$\Rightarrow \vec{b}+\vec{c} = (2+\lambda)\hat{i}+6\hat{j}-2\hat{k}$

The unit vector along $\vec{b}+\vec{c}$ is $\vec{p} = \frac{\vec{b}+\vec{c}}{|\vec{b}+\vec{c}|}$

$= \frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(2+\lambda)^2+6^2+(-2)^2}} = \frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{\lambda^2+4\lambda+44}}$

Also, $\vec{a}.\vec{p} = 1$ (Given)

$\Rightarrow \frac{(2+\lambda)+6-2}{\sqrt{\lambda^2+4\lambda+44}} = 1$

$\Rightarrow \sqrt{\lambda^2+4\lambda+44} = \lambda+6$

$\Rightarrow \quad \lambda^2 + 4\lambda + 44 = \lambda^2 + 12\lambda + 36$

$\Rightarrow \quad 8\lambda = 8 \Rightarrow \lambda = 1$

$\therefore$ The required unit vector

$\vec{p} = \frac{(2+1)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{1+4+44}} = \frac{1}{7}(3\hat{i}+6\hat{j}-2\hat{k})$.

**25.** Two non zero vectors are parallel if and only if their cross product is zero vector.

So, we have to prove that cross product of $\vec{a}-\vec{d}$ and $\vec{b}-\vec{c}$ is zero vector.

Now, $(\vec{a}-\vec{d})\times(\vec{b}-\vec{c}) = (\vec{a}\times\vec{b}) - (\vec{a}\times\vec{c}) - (\vec{d}\times\vec{b}) + (\vec{d}\times\vec{c})$

Since, it is given that $\vec{a}\times\vec{b} = \vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c} = \vec{b}\times\vec{d}$

And, $\vec{d}\times\vec{b} = -\vec{b}\times\vec{d}$, $\vec{d}\times\vec{c} = -\vec{c}\times\vec{d}$

$\therefore \quad (\vec{a}-\vec{d})\times(\vec{b}-\vec{c}) = (\vec{c}\times\vec{d}) - (\vec{b}\times\vec{d}) + (\vec{b}\times\vec{d}) - (\vec{c}\times\vec{d}) = \vec{0}$

Hence, $\vec{a}-\vec{d}$ is parallel to $\vec{b}-\vec{c}$, where $\vec{a}\neq\vec{d}$ and $\vec{b}\neq\vec{c}$.

**26.** Let the required vector be $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$.

Also let, $\vec{a} = \hat{i}-\hat{j}+\hat{k}$, $\vec{b} = 2\hat{i}+\hat{j}-3\hat{k}$ and $\vec{c} = \hat{i}+\hat{j}+\hat{k}$

$\vec{r}\cdot\vec{a} = 4, \ \vec{r}\cdot\vec{b} = 0, \ \vec{r}\cdot\vec{c} = 2$ (Given)

$\Rightarrow \quad x - y + z = 4$ ...(i)

$2x + y - 3z = 0$ ...(ii)

$x + y + z = 2$ ...(iii)

Now (iii) – (i) $\Rightarrow 2y = -2 \Rightarrow y = -1$

From (ii) and (iii)

$2x - 3z - 1 = 0, x + z - 3 = 0 \Rightarrow x = 2, z = 1$

$\therefore$ The required vector is $\vec{r} = 2\hat{i}-\hat{j}+\hat{k}$.

**27.** Here, $\vec{a} = 2\lambda^2\hat{i} + 4\lambda\hat{j} + \hat{k}$ and $b = 7\hat{i} - 2\hat{j} + \lambda\hat{k}$

If $\theta$ is the angle between the vectors $\vec{a}$ and $\vec{b}$, then

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

For $\theta$ to be obtuse, $\cos\theta < 0 \Rightarrow \vec{a}\cdot\vec{b} < 0$

$\Rightarrow \left(2\lambda^2\hat{i} + 4\lambda\hat{j} + \hat{k}\right)\cdot\left(7\hat{i} - 2\hat{j} + \lambda\hat{k}\right) < 0$

$\Rightarrow 2\lambda^2 \cdot 7 + 4\lambda\cdot(-2) + 1\cdot\lambda < 0$

$\Rightarrow 14\lambda^2 - 7\lambda < 0 \Rightarrow \lambda(2\lambda - 1) < 0$

$\Rightarrow$ Either $\lambda < 0$, $2\lambda - 1 > 0$ or $\lambda > 0$, $2\lambda - 1 < 0$

$\Rightarrow$ Either $\lambda < 0, \lambda > \frac{1}{2}$ or $\lambda > 0, \lambda < \frac{1}{2}$

First alternative is impossible.

$\therefore\ \lambda > 0, \lambda < \frac{1}{2}$ *i.e.*, $0 < \lambda < \frac{1}{2}$ *i.e.*, $\lambda \in \left]0, \frac{1}{2}\right[$

**28.** Given, $\Delta ABC$ with vertices

$A(1, 1, 2)$, $B(2, 3, 5)$ and $C(1, 5, 5)$

Now, $\overrightarrow{AB}(2-1)\hat{i} + (3-1)\hat{j} + (5-2)\hat{k}$

$= \hat{i} + 2\hat{j} + 3\hat{k}$,

and $\overrightarrow{AC}(1-1)\hat{i} + (5-1)\hat{j} + (5-2)\hat{k}$

$= 4\hat{j} + 3\hat{k}$.

$$\therefore\quad (\overrightarrow{AB}\times\overrightarrow{AC}) = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 3\\ 0 & 4 & 3\end{vmatrix} = -6\hat{i} - 3\hat{j} + 4\hat{k}$$

Hence, area of $\Delta ABC = \frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|$

$= \frac{1}{2}\sqrt{(-6)^2 + (-3)^2 + 4^2}$

$= \frac{1}{2}\sqrt{61}$ sq. units

**29.** Given, $|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 5$ ...(i)

and $\vec{a}\cdot(\vec{b}+\vec{c}) = 0$, $\vec{b}\cdot(\vec{c}+\vec{a}) = 0$, $\vec{c}\cdot(\vec{a}+\vec{b}) = 0$

$\therefore\quad \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c} + \vec{b}\cdot\vec{c} + \vec{b}\cdot\vec{a} + \vec{c}\cdot\vec{a} + \vec{c}\cdot\vec{b}$

$= 0 + 0 + 0 = 0$

$\Rightarrow 2(\vec{a}\cdot\vec{b}) + 2(\vec{b}\cdot\vec{c}) + 2(\vec{c}\cdot\vec{a}) = 0$ ...(ii)

Now $|\vec{a}+\vec{b}+\vec{c}|^2 = (\vec{a}+\vec{b}+\vec{c})\cdot(\vec{a}+\vec{b}+\vec{c})$

$= (\vec{a}+\vec{b}+\vec{c})\cdot\vec{a} + (\vec{a}+\vec{b}+\vec{c})\cdot\vec{b} + (\vec{a}+\vec{b}+\vec{c})\cdot\vec{c}$

$= \vec{a}\cdot\vec{a} + \vec{b}\cdot\vec{a} + \vec{c}\cdot\vec{a} + \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{b} + \vec{c}\cdot\vec{b} + \vec{a}\cdot\vec{c} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{c}$

$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b}) + 2(\vec{b}\cdot\vec{c}) + 2(\vec{c}\cdot\vec{a})$

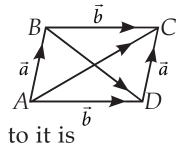
$= 3^2 + 4^2 + 5^2 + 0$ [Using (i) and (ii)]

$= 50$

$\therefore |\vec{a}+\vec{b}+\vec{c}| = 5\sqrt{2}$.

**30.** Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $b = 2\hat{i} + 4\hat{j} - 5\hat{k}$

Then diagonal $\overline{AC}$ of the parallelogram is




$\vec{p} = \vec{a} + \vec{b}$

$= \hat{i} + 2\hat{j} + 3\hat{k} + 2\hat{i} + 4\hat{j} - 5\hat{k}$

$= 3\hat{i} + 6\hat{j} - 2\hat{k}$

Therefore unit vector parallel to it is

$$\frac{\vec{p}}{|\vec{p}|} = \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{9+36+4}} = \frac{1}{7}(3\hat{i} + 6\hat{j} - 2\hat{k})$$

Now, diagonal $\overline{BD}$ of the parallelogram is

$\vec{p}' = \vec{b} - \vec{a} = 2\hat{i} + 4\hat{j} - 5\hat{k} - \hat{i} - 2\hat{j} - 3\hat{k} = \hat{i} + 2\hat{j} - 8\hat{k}$

Therefore unit vector parallel to it is

$$\frac{\vec{p}'}{|\vec{p}'|} = \frac{\hat{i} + 2\hat{j} - 8\hat{k}}{\sqrt{1+4+64}} = \frac{1}{\sqrt{69}}(\hat{i} + 2\hat{j} - 8\hat{k})$$

**31.** Given, $\Delta ABC$ with vertices

$A(1,2,3) \equiv \hat{i} + 2\hat{j} + 3\hat{k}$, $B(2,-1,4) \equiv 2\hat{i} - \hat{j} + 4\hat{k}$,

$C(4,5,-1) \equiv 4\hat{i} + 5\hat{j} - \hat{k}$

Now $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \left(2\hat{i} - \hat{j} + 4\hat{k}\right) - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$

$= \hat{i} - 3\hat{j} + \hat{k}$.

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = \left(4\hat{i} + 5\hat{j} - \hat{k}\right) - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$

$= 3\hat{i} + 3\hat{j} - 4\hat{k}$.

$$\therefore\quad (\overrightarrow{AB}\times\overrightarrow{AC}) = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -3 & 1\\ 3 & 3 & -4\end{vmatrix} = 9\hat{i} + 7\hat{j} + 12\hat{k}$$

Hence, area of $\Delta ABC$

$= \frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}| = \frac{1}{2}\left|9\hat{i} + 7\hat{j} + 12\hat{k}\right|$

$= \frac{1}{2}\sqrt{9^2 + 7^2 + 12^2} = \frac{1}{2}\sqrt{81 + 49 + 144}$

$= \frac{1}{2}\sqrt{274}$ sq. units

**32.** Here, $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}, \vec{b} = 2\hat{i} + \hat{j}, \vec{c} = 3\hat{i} - 4\hat{j} - 5\hat{k}$

$\therefore\ \vec{a} - \vec{b} = (\hat{i} + 2\hat{j} + \hat{k}) - (2\hat{i} + \hat{j}) = -\hat{i} + \hat{j} + \hat{k}$

$\vec{c} - \vec{b} = (3\hat{i} - 4\hat{j} - 5\hat{k}) - (2\hat{i} + \hat{j}) = \hat{i} - 5\hat{j} - 5\hat{k}$

Vector perpendicular to both $\vec{a} - \vec{b}$ and $\vec{c} - \vec{b}$ is

$$(\vec{a} - \vec{b})\times(\vec{c} - \vec{b}) = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & 1 & 1\\ 1 & -5 & -5\end{vmatrix}$$

$= (-5+5)\hat{i} - (5-1)\hat{j} + (5-1)\hat{k} = -4\hat{j} + 4\hat{k}$

$\therefore$ Unit vector perpendicular to both $\vec{a} - \vec{b}$ and $\vec{c} - \vec{b}$

$$= \frac{-4\hat{j} + 4\hat{k}}{\left|-4\hat{j} + 4\hat{k}\right|} = \frac{-4\hat{j} + 4\hat{k}}{\sqrt{(-4)^2 + 4^2}} = \frac{-4\hat{j} + 4\hat{k}}{4\sqrt{2}} = \frac{1}{\sqrt{2}}(-\hat{j} + \hat{k}).$$

**33.** We have $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}+2\hat{j}+3\hat{k}$

Let $\vec{r}=\vec{a}+\vec{b}=2\hat{i}+3\hat{j}+4\hat{k}$

and $\vec{p}=\vec{a}-\vec{b}=-\hat{j}-2\hat{k}$

A unit vector perpendicular to both $\vec{r}$ and $\vec{p}$ is given as $\pm\dfrac{\vec{r}\times\vec{p}}{|\vec{r}\times\vec{p}|}$.

Now, $\vec{r}\times\vec{p}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 4\\ 0 & -1 & -2\end{vmatrix}=-2\hat{i}+4\hat{j}-2\hat{k}$

So, the required unit vector is

$$=\pm\frac{\left(-2\hat{i}+4\hat{j}-2\hat{k}\right)}{\sqrt{(-2)^2+4^2+(-2)^2}}=\mp\frac{\left(\hat{i}-2\hat{j}+\hat{k}\right)}{\sqrt{6}}.$$

**34.** Given, $\hat{a}+\hat{b}=\hat{c}$

$\Rightarrow (\hat{a}+\hat{b})\cdot(\hat{a}+\hat{b})=\hat{c}\cdot\hat{c}$

$\Rightarrow \hat{a}\cdot\hat{a}+\hat{a}\cdot\hat{b}+\hat{b}\cdot\hat{b}+\hat{b}\cdot\hat{a}=\hat{c}\cdot\hat{c}$

$\Rightarrow 1+\hat{a}\cdot\hat{b}+1+\hat{a}\cdot\hat{b}=1$

$\Rightarrow 2\hat{a}\cdot\hat{b}=-1$ ...(i)

Now, $(\hat{a}-\hat{b})^2=(\hat{a}-\hat{b})\cdot(\hat{a}-\hat{b})$

$=\hat{a}\cdot\hat{a}-\hat{a}\cdot\hat{b}-\hat{b}\cdot\hat{a}+\hat{b}\cdot\hat{b}=1-\hat{a}\cdot\hat{b}-\hat{a}\cdot\hat{b}+1$

$=2-2\hat{a}\cdot\hat{b}=2-(-1)$ [Using(i)]

$=3$

$\therefore\ |\hat{a}-\hat{b}|=\sqrt{3}$

**35.** Given $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{j}-\hat{k}$

Let $\vec{c}=x\hat{i}+y\hat{j}+z\hat{k}$

Now we have, $\vec{a}\times\vec{c}=\vec{b}$

$\Rightarrow (\hat{i}+\hat{j}+\hat{k})\times(x\hat{i}+y\hat{j}+z\hat{k})=\hat{j}-\hat{k}$

$\Rightarrow \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ x & y & z\end{vmatrix}=\hat{j}-\hat{k}$

$\Rightarrow \hat{i}(z-y)-\hat{j}(z-x)+\hat{k}(y-x)=\hat{j}-\hat{k}$

$\Rightarrow z-y=0,\ x-z=1$ and $y-x=-1$

$\Rightarrow y=z,\ x-z=1,\ x-y=1$ .....(i)

Also, we have $\vec{a}\cdot\vec{c}=3$

$\Rightarrow (\hat{i}+\hat{j}+\hat{k})\cdot(x\hat{i}+y\hat{j}+z\hat{k})=3$

$\Rightarrow x+y+z=3$

$\Rightarrow x+x-1+x-1=3$ [Using (i)]

$\Rightarrow 3x-2=3 \Rightarrow x=\dfrac{5}{3},\ y=\dfrac{2}{3},\ z=\dfrac{2}{3}$

Hence, $\vec{c}=\dfrac{5}{3}\hat{i}+\dfrac{2}{3}\hat{j}+\dfrac{2}{3}\hat{k}$

**36.** $|\vec{a}|=|\vec{b}|=|\vec{c}|$ (Given) ...(i)

and $\vec{a}\cdot\vec{b}=0,\ \vec{b}\cdot\vec{c}=0,\ \vec{c}\cdot\vec{a}=0$ ...(ii)

Let $(\vec{a}+\vec{b}+\vec{c})$ be inclined to vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ by angles α, β and γ respectively. Then

$$\cos\alpha=\frac{(\vec{a}+\vec{b}+\vec{c})\cdot\vec{a}}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}=\frac{\vec{a}\cdot\vec{a}+\vec{b}\cdot\vec{a}+\vec{c}\cdot\vec{a}}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}$$

$=\dfrac{|\vec{a}|^2+0+0}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}$ [Using (ii)]

$=\dfrac{|\vec{a}|}{|\vec{a}+\vec{b}+\vec{c}|}$ ...(iii)

Similarly, $\cos\beta=\dfrac{|\vec{b}|}{|\vec{a}+\vec{b}+\vec{c}|}$ ...(iv)

and $\cos\gamma=\dfrac{|\vec{c}|}{|\vec{a}+\vec{b}+\vec{c}|}$ ...(v)

From (i), (iii), (iv) and (v), we get

$\cos\alpha=\cos\beta=\cos\gamma \Rightarrow \alpha=\beta=\gamma$

Hence, the vector $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to the vector $\vec{a}$, $\vec{b}$ and $\vec{c}$.

Also the angle between them is given as

$$\alpha=\cos^{-1}\left(\frac{|\vec{a}|}{|\vec{a}+\vec{b}+\vec{c}|}\right),\ \beta=\cos^{-1}\left(\frac{|\vec{b}|}{|\vec{a}+\vec{b}+\vec{c}|}\right),$$

$$\gamma=\cos^{-1}\left(\frac{|\vec{c}|}{|\vec{a}+\vec{b}+\vec{c}|}\right)$$

**37.** We have, $A(2\hat{i}-\hat{j}+\hat{k})$, $B(\hat{i}-3\hat{j}-5\hat{k})$ and $C(3\hat{i}-4\hat{j}-4\hat{k})$

Then, $\overrightarrow{AB}=(1-2)\hat{i}+(-3+1)\hat{j}+(-5-1)\hat{k}$

$=-\hat{i}-2\hat{j}-6\hat{k}$

$\overrightarrow{AC}=(3-2)\hat{i}+(-4+1)\hat{j}+(-4-1)\hat{k}=\hat{i}-3\hat{j}-5\hat{k}$

and $\overrightarrow{BC}=(3-1)\hat{i}+(-4+3)\hat{j}+(-4+5)\hat{k}=2\hat{i}-\hat{j}+\hat{k}$

Now angle between $\overrightarrow{AC}$ and $\overrightarrow{BC}$ is given by

$$\cos\theta=\frac{(\overrightarrow{AC})\cdot(\overrightarrow{BC})}{|\overrightarrow{AC}||\overrightarrow{BC}|}=\frac{2+3-5}{\sqrt{1+9+25}\,.\,\sqrt{4+1+1}}$$

$\Rightarrow \cos\theta=0 \Rightarrow BC\perp AC$

So, $A$, $B$, $C$ are vertices of right angled triangle.

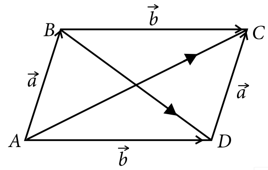
Now area of $\Delta ABC=\dfrac{1}{2}|\overrightarrow{AC}\times\overrightarrow{BC}|$

$$=\frac{1}{2}\left\|\begin{matrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -3 & -5\\ 2 & -1 & 1\end{matrix}\right\|=\frac{1}{2}|(-3-5)\hat{i}-(1+10)\hat{j}+(-1+6)\hat{k}|$$

$= \frac{1}{2} |-8\hat{i} - 11\hat{j} + 5\hat{k}|$

$= \frac{1}{2}\sqrt{64+121+25} = \frac{\sqrt{210}}{2}$ sq. units.

**38.** Let $\vec{a} = 2\hat{i} - 4\hat{j} - 5\hat{k}$ and $\vec{b} = 2\hat{i} + 2\hat{j} + 3\hat{k}$




Then diagonal $\overline{AC}$ of the parallelogram is

$\vec{p} = \vec{a} + \vec{b}$

$= 2\hat{i} - 4\hat{j} - 5\hat{k} + 2\hat{i} + 2\hat{j} + 3\hat{k} = 4\hat{i} - 2\hat{j} - 2\hat{k}$

Therefore, unit vector parallel to it is

$\frac{\vec{p}}{|\vec{p}|} = \frac{4\hat{i} - 2\hat{j} - 2\hat{k}}{\sqrt{16+4+4}} = \frac{2\hat{i} - \hat{j} - \hat{k}}{\sqrt{6}}$

Now, diagonal $\overline{BD}$ of the parallelogram is

$\vec{p}' = \vec{b} - \vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k} - 2\hat{i} + 4\hat{j} + 5\hat{k} = 6\hat{j} + 8\hat{k}$

Therefore, unit vector parallel to it is

$\frac{\vec{p}'}{|\vec{p}'|} = \frac{6\hat{j} + 8\hat{k}}{\sqrt{36+64}} = \frac{6\hat{j} + 8\hat{k}}{10} = \frac{3\hat{j} + 4\hat{k}}{5}$

Now, $\vec{p} \times \vec{p}' = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -2 & -2 \\ 0 & 6 & 8 \end{vmatrix}$

$= \hat{i}(-16+12) - \hat{j}(32-0) + \hat{k}(24-0)$

$= -4\hat{i} - 32\hat{j} + 24\hat{k}$

$\therefore$ Area of parallelogram $= \frac{|\vec{p} \times \vec{p}'|}{2}$

$= \frac{\sqrt{16+1024+576}}{2} = 2\sqrt{101}$ sq. units.

**39.** Here $\vec{a} = 3\hat{i} - \hat{j}$, $\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$

We have to express : $\vec{b} = \vec{b}_1 + \vec{b}_2$, where

$\vec{b}_1 \,||\, \vec{a}$ and $\vec{b}_2 \perp \vec{a}$

Let $\vec{b}_1 = \lambda\vec{a} = \lambda(3\hat{i} - \hat{j})$ and $\vec{b}_2 = x\hat{i} + y\hat{j} + z\hat{k}$

Now $\vec{b}_2 \perp \vec{a} \Rightarrow \vec{b}_2 \cdot \vec{a} = 0$

$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (3\hat{i} - \hat{j}) = 0$

$\Rightarrow \quad 3x - y = 0$ ...(i)

Now, $\vec{b} = \vec{b}_1 + \vec{b}_2$

$\Rightarrow 2\hat{i} + \hat{j} - 3\hat{k} = \lambda(3\hat{i} - \hat{j}) + (x\hat{i} + y\hat{j} + z\hat{k})$

On comparing, we get

$\left.\begin{array}{l} 2 = 3\lambda + x \\ 1 = -\lambda + y \end{array}\right] \Rightarrow x + 3y = 5$ ...(ii)

and $-3 = z \Rightarrow z = -3$

Solving (i) and (ii), we get $x = \frac{1}{2}, y = \frac{3}{2}$

$\therefore 1 = -\lambda + y \Rightarrow 1 = -\lambda + \frac{3}{2} \Rightarrow \lambda = \frac{1}{2}$

Hence, $\vec{b}_1 = \lambda(3\hat{i} - \hat{j}) = \frac{3}{2}\hat{i} - \frac{1}{2}$

and $\vec{b}_2 = \frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}$

**40.** Given, position vector of $A = \hat{i} + \hat{j} + \hat{k}$

Position vector of $B = 2\hat{i} + 5\hat{j}$

Position vector of $C = 3\hat{i} + 2\hat{j} - 3\hat{k}$

Position vector of $D = \hat{i} - 6\hat{j} - \hat{k}$

$\therefore \quad \overline{AB} = (2\hat{i} + 5\hat{j}) - (\hat{i} + \hat{j} + \hat{k}) = \hat{i} + 4\hat{j} - \hat{k}$ and

$\overline{CD} = (\hat{i} - 6\hat{j} - \hat{k}) - (3\hat{i} + 2\hat{j} - 3\hat{k}) = -2\hat{i} - 8\hat{j} + 2\hat{k}$

Now $|\overline{AB}| = \sqrt{(1)^2 + (4)^2 + (1)^2} = \sqrt{18}$

$|\overline{CD}| = \sqrt{(-2)^2 + (-8)^2 + (2)^2} = \sqrt{4+64+4}$

$= \sqrt{72} = 2\sqrt{18}$

Let $\theta$ be the angle between $\overline{AB}$ and $\overline{CD}$.

$\therefore \cos\theta = \frac{\overline{AB} \cdot \overline{CD}}{|\overline{AB}||\overline{CD}|} = \frac{(\hat{i} + 4\hat{j} - \hat{k}) \cdot (-2\hat{i} - 8\hat{j} + 2\hat{k})}{(\sqrt{18})(2\sqrt{18})}$

$= \frac{-2-32-2}{36} = \frac{-36}{36} = -1$

$\Rightarrow \quad \cos\theta = -1 \quad \Rightarrow \quad \theta = \pi$

Since, angle between $\overline{AB}$ and $\overline{CD}$ is 180°.

$\therefore \quad \overline{AB}$ and $\overline{CD}$ are collinear.